Lesson 9

Work – Energy

"Work-Energy" relation is a very important concept. This is another thing that we use those terms in our daily lives, but because lots of people don't know exact definitions, the terms are used vaguely and sometimes are misused. However, you recognize by now that you are not allowed to misuse the terms in this class or other classes in Physics.

Energy – Ability to do work.

Work – Change of Kinetic Energy.

We have not defined Kinetic Energy yet, but you can think it as "moving" energy. Therefore, kinetic energy is associated with "speed". The relation between "outside force" and "work" follows this way.

1. To change kinetic energy, speed needs to be changed.

2. To change speed, acceleration is needed.

3. To acquire acceleration, non-zero net outside force is needed.

4. Furthermore, the direction of motion and outside force have to be parallel (perpendicular direction changes direction, not speed).

According to the definition and relation between the force and kinetic energy (be able to describe them well), the mathematical expression becomes:

$W = \int \vec{F} \cdot d\vec{r}$, where $\vec{F}$ is an outside force and $d\vec{r}$ is the direction of motion. Hence, the kinetic energy is derived this way:

$$W = \int \vec{F} \cdot d\vec{r}$$

$$= \int m\vec{a} \cdot d\vec{r} \text{ (using only parallel component of a and r)}$$

$$= \int m \frac{dv}{dt} \cdot v\, dt \quad \left(\text{since } a = \frac{dv}{dt} \text{ and } v = \frac{dr}{dt}\right)$$

$$= \int_{v_i}^{v_f} mv \cdot dv$$

$$= \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

If $v_i = 0$ m/sec, then, the kinetic energy of a moving object is $\frac{1}{2} mv_f^2$. Notice that kinetic energy is proportional to the square of speed. Also, the resultant of a dot product is a scalar, not a vector. Hence, energy does not have any direction.

Units of energy

From the equation we just derived ($\frac{1}{2}$ mv_f^2), or $\int \vec{F} \cdot d\vec{r}$ will give the units of Nm. However, there are two vector multiplications (dot product and cross product) and they results show totally different physical meanings. Although as far as units are concerned, both products give (Nm), but we need differentiate so that we can tell which operation has been taken the place by looking at the units. We promise the units of energy (and work) to be J (Joules) and units of the cross product $\int \vec{F} \times d\vec{r}$ to be Nm. If "cm" and "grams" are used ($g\frac{cm^2}{sec^2}$), the energy units is called "erg". There are lots of types of energy, but we will focus four different types of energy, Kinetic, Frictional, Spring, and Gravitational Potential Energy. You should be able to derive all of these types. First of all, consider if a force is doing a positive or a negative work.

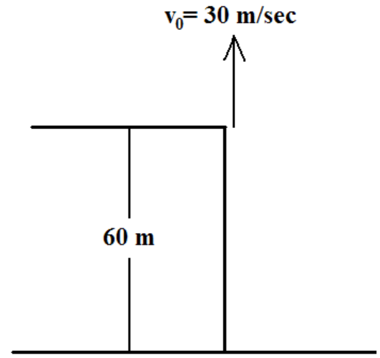
Frictional Energy ($F_{fric} = \mu_k N$)

Spring Energy ($F_{spring} = -kx$), where k is Spring constant

Gravitational Potential Energy ($F_{Grav} = G\frac{m_1 m_2}{r^2}$) , where G is Gravitational constant ($6.6726 \times 10^{-11} Nm^2/kg^2$)

Conservation of energy –

Using "conservation of energy" idea is useful and easily done IF THERE IS NO TIME INVOLVED. If you have to calculate time, conservation of energy will not work



Example 1 (We did this using motion equations)

An object is ejected at 30 m/sec from the top of a 60m-cliff.

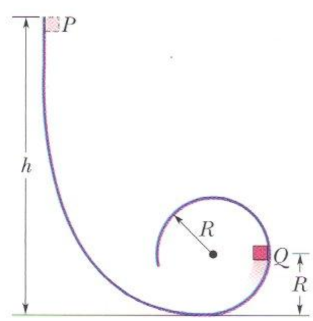
Calculate: (a) Maximum height, (b) Velocity when it comes back to the initial position, and (c) Impact velocity when it hits the ground

Example 2

A ball is dropped from a height of 2.2 m and rebounds to a height of 1.9 m above the floor. Assume the ball was in contact with the floor for 96 ms (milliseconds). Determine the average acceleration (both magnitude and direction) of the ball during the contact with the floor.

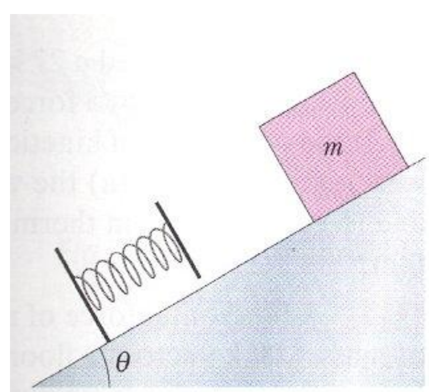
Example 3

In the figure shown, a small block of mass m= 0.032 kg can slide along the frictionless loop-the-loop, with radius R= 12 cm. The block is released from rest at point P, at height h = 5.0 R above the bottom of the loop. How much work does the gravitational Force do on the block as the block travels from point P to (a) point Q and (b) the top of the loop? If the gravitational potential energy of the block-earth system is taken to be zero at the bottom of the loop, what is that potential energy when the block is at (c) point P, (d) at point Q, and (e) at the top of the loop? (f) If, instead of merely being released, the block is given some initial speed downward along the track, do the answers to (a) through (e) increase, decrease, or remain the same?



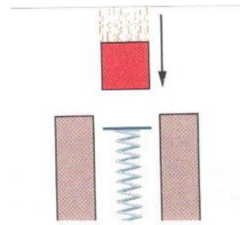
Example 4

In the figure, a block of mass m = 12 kg is released from rest on a frictionless incline of angle θ = 30.0º. Below the block is a spring that can be compressed 2.0 cm by a Force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm. (a) How far does the block move down the incline from its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring?



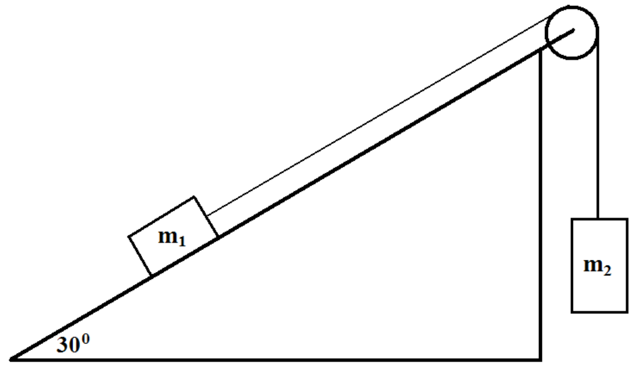
Example 5

A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of k = 2.5 N/cm. The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping. While the spring is being compressed, what work is done on the block by (a) the gravitational force on it and (b) the spring force? (c) What is the speed of the block just before it hits the spring? (Assume that friction is negligible.) (d) If the speed at impact is doubled, what is the maximum compression of the spring?



Example 6

Two blocks are connected by a massless string. m_1 is on an inclined plane and m_2 is suspended. Initially, m_1 is moving upward at 15 m/sec. m_1 = 10 kg, m_2 = 6 kg, and $\mu_s = \mu_k = 0.01$, what is the velocity of m_1 when it slides up 5 m on the incline?



Definition Math Translation

Power –

Example 1

A 0.30 kg ladle sliding on a horizontal frictionless surface is attached to one end of a horizontal spring (k = 500N/m) whose other end is fixed. The ladle has a kinetic energy of 10 J as it passes through its equilibrium position (the point at which the spring force is zero). (a) At what rate is the spring doing work on the ladle as the ladle passes through its equilibrium position? (b) At what rate is the spring doing work on the ladle when the spring is compressed 0.10 m and the ladle is moving away from the equilibrium position?