Lesson 12

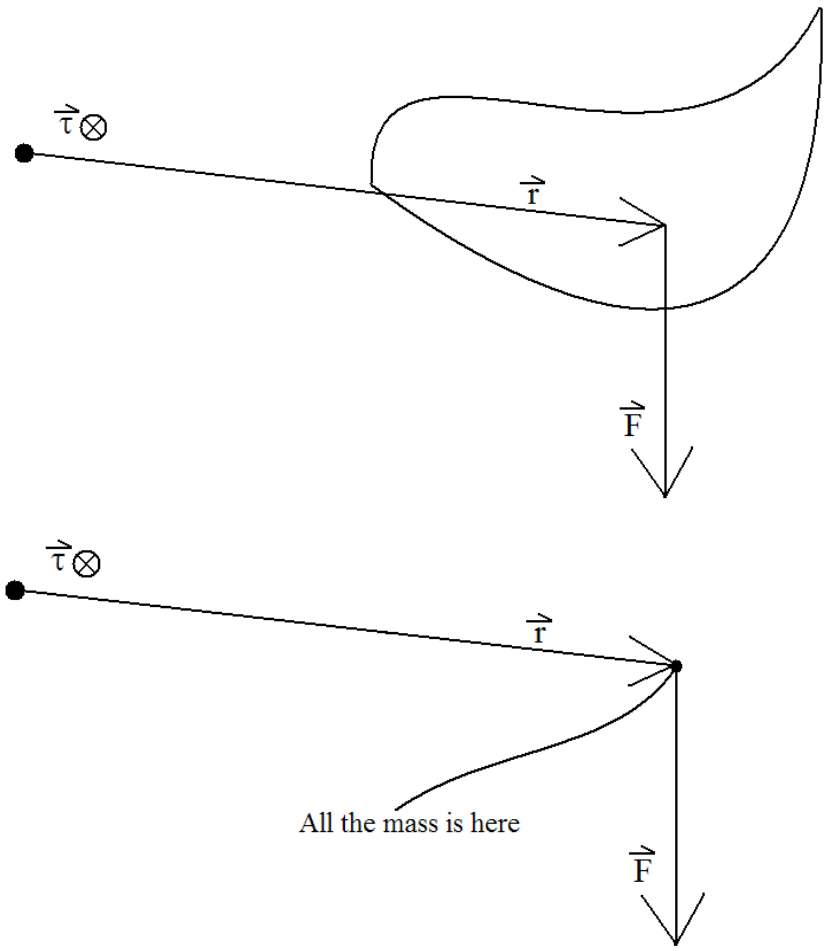
Center of Mass

Center of Mass – A point where a non-point mass object can be treated as a point mass.

For a small object, the center of mass and the center of gravity are at the same point. However, they are not the same when the object is large. In physics, we want to use "center of mass", not "center of gravity".

How to calculate Center of Mass

The idea is that an object should produce the same amount of torque (with respect to the origin of your choice) with a real shape of the object or the all of its mass concentrated at the center of mass.



These situations above create same torque if all mass is put at the center of mass.

You are going to learn how to calculate center of mass, not because you will be calculating it when you get a real job, but you are practicing how to use calculus. This method is used everywhere. It will make 230 much easier if you learn this method now.

Steps

1. Draw a big diagram with the origin clearly indicated.
2. Take a small chunk called “dm”.
3. Calculate torque created by dm WRT the origin. ($d\vec{\tau} = \vec{r} \times dm \cdot \vec{g}$: Notice that the order of the cross product is very important.)
4. Rewrite the equation in terms of one variable.
5. Calculate the total torque created by the object (by integration), which has to be equal to R×Mg, where R is the distance from the origin to the center of mass.

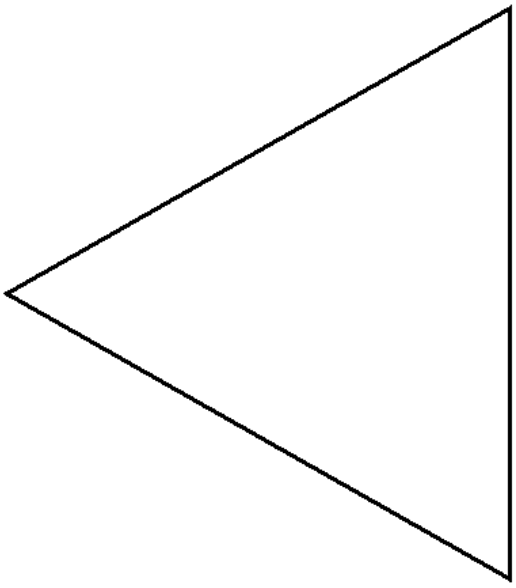
The steps are always the same for any shape and the only way you can do well is practice.

Example 1 – Thin Rod

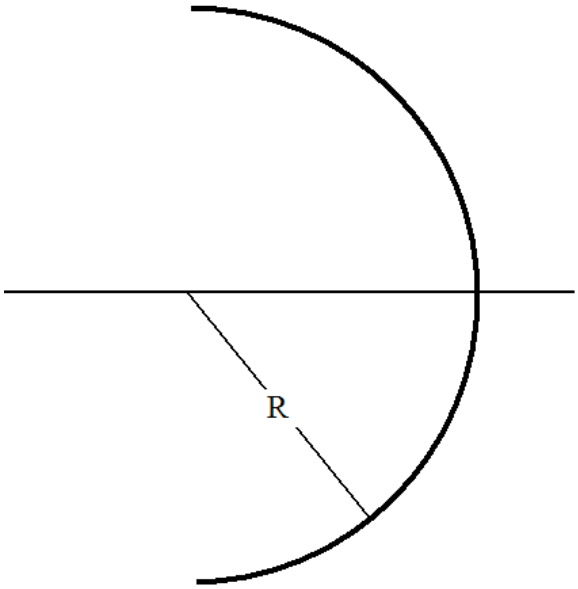
Example 2 – Equilateral Triangle

(a) You can solve for the center of mass geometrically.
(b) Confirm your result by following the new steps using calculus.

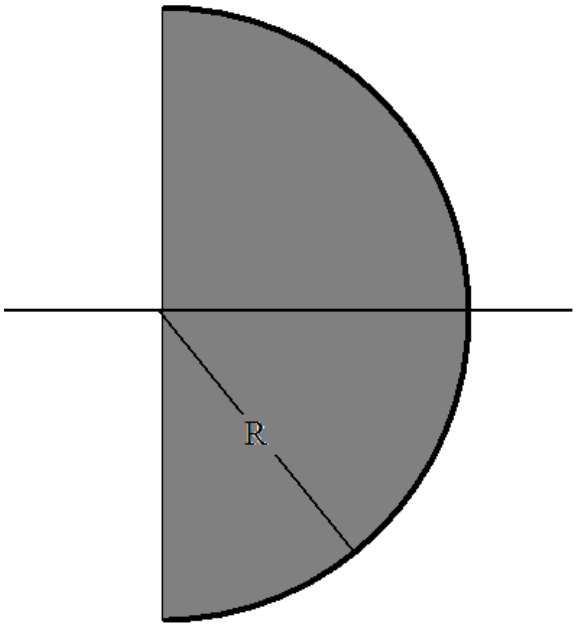
Step1 (done for you)



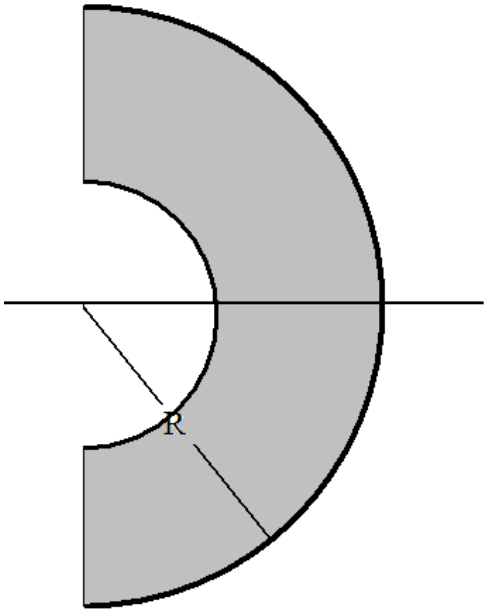
Example 3 – Half Circle



Example 4 – Half Disc



Example 5 – Half Washer



Example 6 – Solid Cone