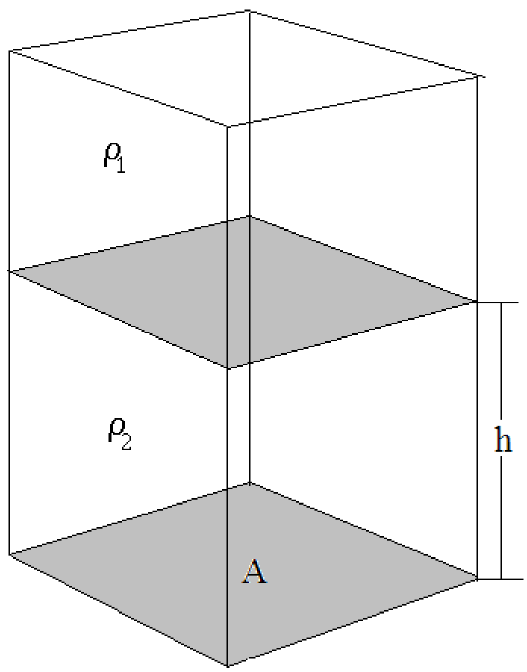
FLUID DYNAMICS

Pressure

$\rho = M/vol$

$p_r = F/A = mg/A$ (for fluid) $= \frac{\rho \cdot vol \cdot g}{A}$ unit:≡ 1Pa, 1ATM = 101.3kPa = 1013 mb = 76.0 cm Hg



Total pressure at the bottom; $p_{r1} + p_{r2}$

$$p_{rT} = \frac{\rho_1 . vol_1 \cdot g}{A} + \frac{\rho_2 . vol_2 \cdot g}{A}$$

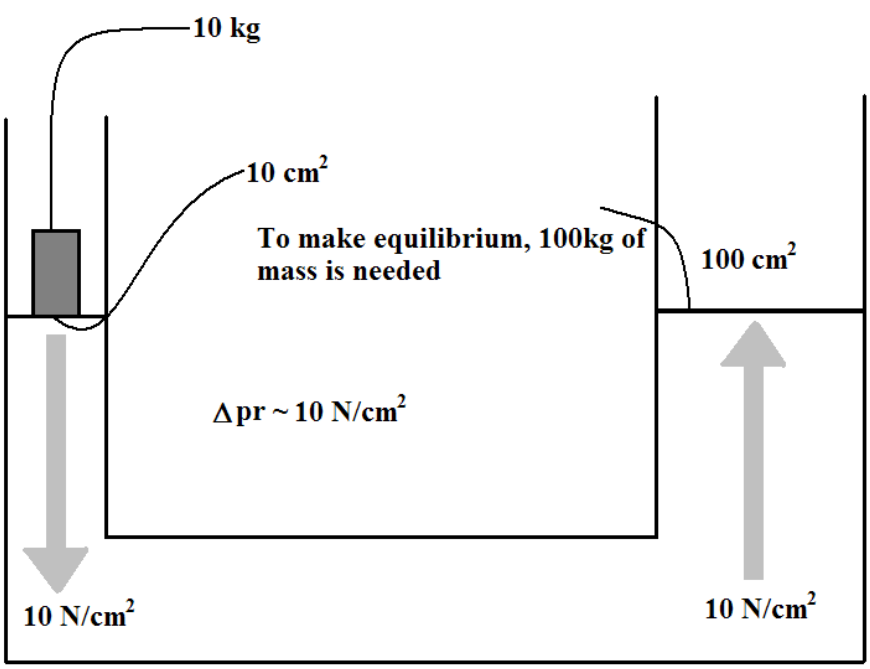
$$= \frac{\rho_1 . vol_1 \cdot g}{A} + \frac{\rho_2 \cdot Ah \cdot g}{A}$$

If $\frac{\rho_1 . vol_1 \cdot g}{A}$ is atmosphere and pr_2 is under the water,

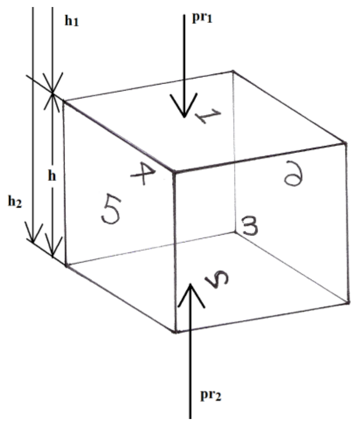
$$= pr_{,0} + \rho_{water} hg$$

Pascal's Principle

Δpr to the enclosed fluid is transmitted to every part of the body.



Buoyant Force



Horizontal direction: $\sum pr_x = 0$

Vertical direction: $\Delta pr = \overrightarrow{pr_1} + \overrightarrow{pr_2}$

$= -\rho g h_1 + \rho g h_2$

$= \rho g(-h_1 + h_2)$

$= \rho g(h)$

h: height of the object in the liquid

$\therefore\ \Delta F = A \cdot \Delta pr$

$= A \cdot \rho g(h) = \rho g \cdot (Ah)$

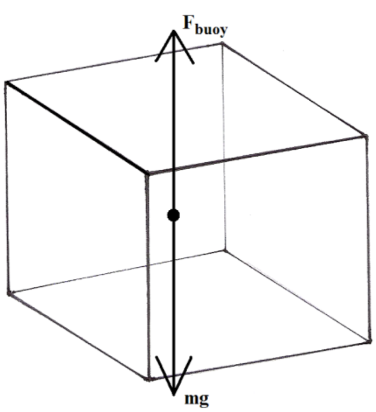
$= \rho g \cdot (vol)$

Since "ρ" is the density of liquid, this extra "upward" force is same as;

(mass of the liquid displaced by a body (ρ · vol)) x g = weight of the liquid displaced by the body

This upward force is called a "<u>buoyant force</u>".

Net force in a liquid



$\sum F_{buoy} - F_{obj}$

$= \rho_{liquid} \cdot g \cdot (vol)_{displaced} - \rho_{obj} \cdot g \cdot (vol)_{obj}$

Since, $(vol)_{displaced} = (vol)_{obj}$

$= (\rho_{liquid} - \rho_{obj}) \cdot g \cdot (vol)_{obj}$

This result shows us the net force is depending upon the density comparison between the liquid and the object.

If,

$\rho_{liquid} > \rho_{object}$, the object will float

$\rho_{liquid} = \rho_{object}$, the object will stay

$\rho_{liquid} < \rho_{object}$, the object will sink

What if the density of the object is less than the density of liquid? How much will it submerge in the liquid? Of course the answer is "it sinks until the net force becomes zero = until the (total) weight of the object is equal to the weight of the liquid displaced by the object." In this case, $(\text{vol})_{\text{displaced}} \neq (\text{vol})_{\text{obj}}$.

$$\sum F_{\text{buoy}} - F_{\text{obj}} = 0$$

$$= \rho_{\text{liquid}} \cdot g \cdot (\text{vol})_{\text{displaced}} - \rho_{\text{obj}} \cdot g \cdot (\text{vol})_{\text{obj}} = 0$$

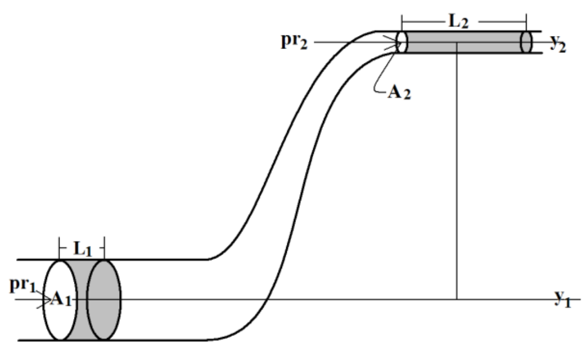
$$(\text{vol})_{\text{displaced}} = \frac{\rho_{\text{obj}} \cdot g \cdot (\text{vol})_{\text{obj}}}{\rho_{\text{liquid}} \cdot g}$$

$$= \frac{\rho_{\text{obj}} \cdot (\text{vol})_{\text{obj}}}{\rho_{\text{liquid}}}$$

$$= \frac{\text{Mass of the object}}{\rho_{\text{liquid}}}$$

Bernoulli's equation

Work done onto the system from A_1side



W $= F \cdot r$

$= F_1 \cdot r_1$

$= pr_1 A_1 \cdot r_1$

$E_i = E_f$

$pr_1 A_1 \cdot r_1 = \Delta W + \Delta PE + \Delta KE$

$= pr_2 A_2 \cdot r_2 + mg(y_2 - y_1) + \frac{1}{2} m(v_2^2 - v_1^2)$

Since $A_1 r_1 = A_2 r_2$ = volume moved $= \frac{m}{\rho}$

$pr_1 \frac{m}{\rho} = pr_2 \frac{m}{\rho} + mg(y_2 - y_1) + \frac{1}{2} m(v_2^2 - v_1^2)$ (Divided by "m" and multiply by "ρ")

$pr_1 = pr_2 + \rho g(y_2 - y_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$ (Gather all "1's on the LHS)

$$\boxed{pr_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = pr_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = \text{constant}}$$

BREAK TIME READING: APPLICATION OF PHYSICS FOR PHUN

Scaling - ratio of volume to surface area

This indicates that larger object has a smaller surface area to its volume.

Example: 1 cm^3 vs. 2 cm^3 cubes

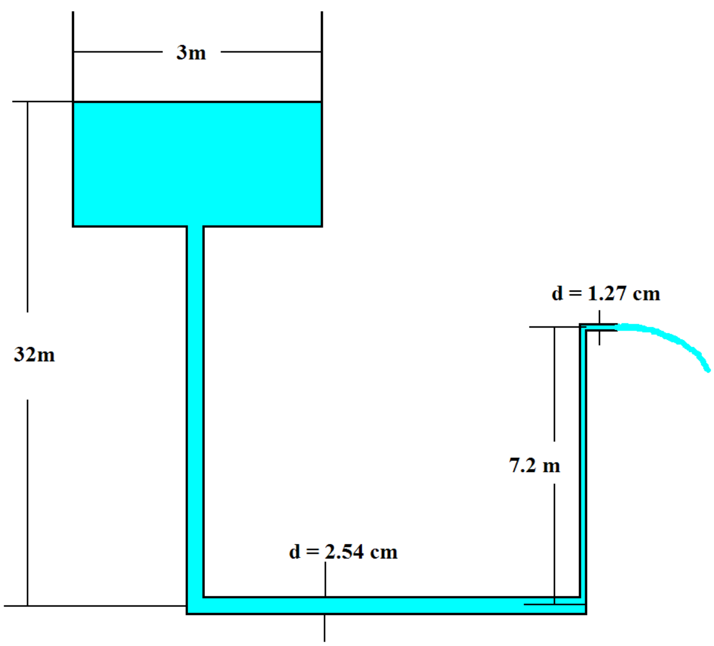
$$\frac{\text{surface area}}{\text{volume}} = \frac{6\ (1\ \text{cm} \times 1\ \text{cm})}{1\ \text{cm} \times 1\ \text{cm} \times 1\ \text{cm}} = 6 \text{ (don't worry about the units)}$$

$$\frac{\text{surface area}}{\text{volume}} = \frac{6\ (2\ \text{cm} \times 2\ \text{cm})}{2\ \text{cm} \times 2\ \text{cm} \times 2\ \text{cm}} = 3$$

So, if you double a linear dimension, the ratio halves! You can explain many things with "scaling".

1. Why does a kindling catch fire more easily than a log?
2. Why does a baby loses water faster than an adult?
3. Which one produces more peelings, 10 lb of small potatoes or 10 lb of large potatoes? (Okay, if you don't like potatoes, you can use apples.)
4. Why does an elephant have large ears compared to human ears?
5. Which one cooks faster in an oven, a 1 lb meat ball or ten – 1/10 of a pound meat ball?
6. Which one cooks faster in an oven, a Bundt cake or the same amount poured in several cupcakes?

Example 1.



Rate of water flow = 0.0025 m^3/sec

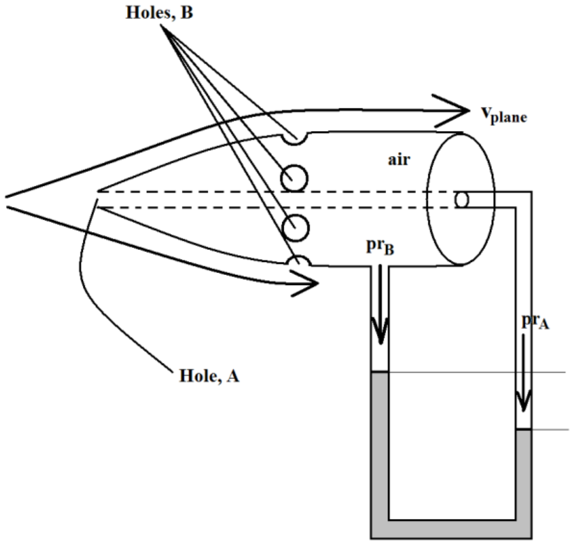
(~ 2/3 gallon/sec)

(a) Pressure at the horizontal pipe?

(Answer: If water is flowing, 4.03 X 10^5 pa and if not, 4.15 X 10^5 pa)

(b) Pressure at 7.2 m high? (Answer: 1.49 X 10^5 pa)

Example 2 – Pitot tube



(Real Pitot tube – you can see a small hole in the tube as well)

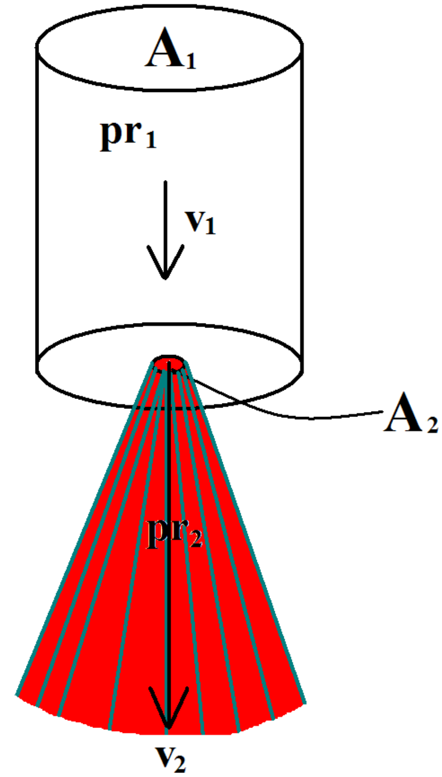
Calculate air speed of a plane.

At A, air becomes stagnant – $V_A = 0$

At B, (presumably equals air speed V of the air plane) – $V_B = V_{plane}$

Pressures at surfaces created by due to the potential difference are about the same ($y_1 - y_2 = 0$).

Example 3 – Thrust of a Rocket Engine



$$pr_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = pr_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = \text{constant}$$

Pressures at surfaces created by due to the potential difference are about the same ($y_1 - y_2 = 0$).

$$\frac{1}{2}\rho v_2^2 = (pr_1 - pr_2) + \rho g\,(y_1 - y_2) + \frac{1}{2}\rho v_1^2$$

$y_1 - y_2 \sim 0$ because it is about just before and after the exhaust.

$$\therefore \frac{1}{2}\rho v_2^2 = (pr_1 - pr_2) + \frac{1}{2}\rho v_1^2$$

$$v_2^2 = \frac{2(pr_1 - pr_2)}{\rho} + v_1^2$$

Also, $A_1V_1 = A_2V_2 \rightarrow v_1 = \frac{A_2 v_2}{A_1}$ If $A_2 \ll A_1$, then, $v_2 \gg v_1$

$$\text{so, } v_2^2 \sim \frac{2(pr_1 - pr_2)}{\rho}$$

$$\therefore\ v_2 \sim \sqrt{\frac{2(pr_1 - pr_2)}{\rho}}\ \text{(ejection speed)} \rightarrow 1$$

Thrust = $v_2 \frac{dm}{dt} = v_2 \rho A_2 v_2 = \rho A_2 v_2^2 \leftarrow 1$

$$= \rho A_2 \frac{2(pr_1 - pr_2)}{\rho}$$

$$= 2A_2(pr_1 - pr_2)$$

Conclusion:

The thrust is depending upon the gas pressure difference between inside and outside and the size of the exhaust. pr_2 will be very similar to 1 ATM. Therefore, the explosion of the gas inside of the chamber is a factor for higher thrust. (This also means that how much of pressure the chamber can hold, i.e. the strength of the material is very important as well as how explosive the gas is.) The second factor, A_2 is a tricky one. If A_2 increases it seems like the thrust increases, but the pressure difference decreases. Hence, there is an optimal size of A_2 and the pressure difference to cause the maximum thrust for a certain explosion – the detail results can be achieved only from experiments – once again, if you a pyromania, there is no better job than Rocket Engineering.