When you solve any physics problem, follow the steps.

1. Draw an accurate diagram after you read the problem.
2. Pick a point
3. Draw forces, momentum or any approprate vectors.
4. Write equations about the point (all the equations even if you don't use them all.)
5. Show all of your work.
6. Do not forget units (especially on your final answers!)
7. Do not put an equal sign if they are not equal.

   Ex. Calculate the area of a circle whose diameter is 4.5 cm.

   wrong way: $\frac{4.5\,cm}{2} = (2.25\,cm)^2 \cdot \pi = \underline{\underline{15.9\,cm^2}}$

   correct way: $Area = \pi r^2 = \pi \cdot \left(\frac{4.5\,cm}{2}\right)^2 = \underline{\underline{15.9\,cm^2}}$

8. Carry as many digits as possible (you don't have to write them all) during the calculation. Do not round-off or round-up until the end.

Physics 230

ch. 2. #19, #25, #41, #50, #58, #78, #89, #115

#19. $x = ct^2 - bt^3$

(a) Dimensional Analysis : It is very important. Even though it takes longer, it helps us to see if equations are indeed correct.

$\Rightarrow$ units are very important!

$$\begin{array}{ccccccc} x & = & c\ t^2 & - & b\ t^3 \\ \downarrow & & \downarrow\ \downarrow & & \downarrow\ \downarrow \\ \text{units}\quad m & = & (c)(\text{sec})^2 & - & (b)(\text{sec})^3 \end{array}$$

$$\therefore \begin{cases} m = (c)\,\text{sec}^2 & \Rightarrow \underline{\underline{(c) = m/\text{sec}^2}} \\ m = (b)\,\text{sec}^3 & \Rightarrow \underline{\underline{(b) = m/\text{sec}^3}} \end{cases}$$

(b)

$$x = \underset{(m/\text{sec}^2)}{3}t^2 - \underset{(m/\text{sec}^3)}{2}t^3$$

This is a maximization (or minimization) prob.
To solve this type, we take a derivative of the fcn, and set it equals to zero ($\frac{dx}{dt} = 0$) then check $\frac{dx}{dt}$ (= vel.) before & after the solution. If $\frac{dx}{dt}$ is changing from + to −, then the sol. is max. (If the vel. is changing from positive (going away) to negative (coming back), the peak must be max.)

(c)

$$\frac{dx}{dt} = 2\,(3\ ^{m}/_{sec^2})\,t - 3\,(2\ ^{m}/_{sec^3})\,t^2$$

$$= 6\,t\,(^{m}/_{sec^2}) - 6\,t^2\,(^{m}/_{sec^3})$$

$$6t - 6t^2 = 0$$

$$6t(1-t) = 0$$

$t = 0$ sec & $1$ sec.

| $t$ | $\frac{dx}{dt}$ |
|---|---|
| $t < 0$ sec | $-$ |
| $0$ sec | $0$ |
| $0 < t < 1$ sec | $+$ |
| $1$ sec | $0$ |
| $t > 1$ sec | $-$ |

As you can see, at $t = 1$ sec the particle reaches its max.

(d) Distance − (Total movement)

(e) Displacement − (Distance from the origin)

| t (sec) | (c) Distance (m) | (d) Displacement (m) |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 5 | −4 |
| 3 | 23 | −27 |
| 4 | 53 | −80 |
| | 82 (Total Distance moved from t=0 to 4sec) | |

(f) — (m) $V = \frac{dx}{dt} = 6t(m/sec^2) - 6t^2(m/sec^3)$

$a = \frac{dV}{dt} = 6\,m/sec^2 - 12t(m/sec^3)$

| t(sec) | V(m/sec) | a(m/sec²) |
|---|---|---|
| 1 | 0 | -6 |
| 2 | -12 | -18 |
| 3 | -36 | -30 |
| 4 | -72 | -42 |

Hint: How to start.

#25

$V_0 = 1.5 \times 10^5\,m/sec$

$V_f = 5.7 \times 10^6\,m/sec$

$X = 1.0\,cm = 1.0 \times 10^{-2}\,m$

$a = const$ ——— ①

$V = \int a \cdot dt = at + V_0$ ——— ②

$X = \int V \cdot dt = \int (at + V_0)dt = \frac{1}{2}at^2 + V_0 t + \cancel{X_0}^{0}$ — ③

For any question, write down what you know. If it is an eqn, label it.

Eqn ②, solve for t (Hint: write down what you are going to do → It tells graders that you know what you are doing and it is easier for graders to follow you.)

$V = at + V_0$

$at = V - V_0$

$t = \frac{V - V_0}{a}$ ——— ②'

③ ← ②'

$X = \frac{1}{2}at^2 + V_0 t$

$= \frac{1}{2}a\left(\frac{V - V_0}{a}\right)^2 + V_0\left(\frac{V - V_0}{a}\right)$

$$X = \frac{1}{2} a \frac{(V-V_0)^2}{a^2} + \frac{V_0(V-V_0)}{a}$$

$$= \frac{V^2 - 2VV_0 + V_0^2}{2a} + \frac{2V_0V - 2V_0^2}{2a}$$

$$= \frac{V^2 - V_0^2}{2a}$$

$$\therefore \quad a = \frac{V^2 - V_0^2}{2X}$$

$$= \frac{(5.7\times10^6)^2 - (1.5\times10^5)^2}{2 \cdot 1\times10^{-2}}$$

$$= \underline{\underline{1.62 \times 10^{15}\ m/sec^2}}$$

\# 41

It is a common sense to use "X" for a horizontal distance and "y" for a vertical distance.

$$a = -g \text{ ———— } (1)$$

$$V = \int a\,dt = -gt + V_0 \text{ ———— } (2)$$

$$y = \int V\,dt = -\frac{1}{2}gt^2 + V_0t + \cancel{y_0}^{\,0} \text{ — } (3)$$

at its max. height (50m), V=0.

Eqn. (2)

$$0 = -gt + V_0$$

$$t = \frac{V_0}{g} \text{ ———— } (2)'$$

$(3) \leftarrow (2)'$

$$y = -\frac{1}{2}gt^2 + V_0t$$

$$50m = -\frac{1}{2}g\left(\frac{V_0}{g}\right)^2 + V_0\left(\frac{V_0}{g}\right)$$

$$50 = \frac{1}{2}\frac{V_0^2}{g}$$

$$\therefore V_0 = \sqrt{50 \cdot 2 \cdot g}$$

$$= \underline{\underline{31.32 \text{ m/sec}}} \text{ ——————— } ③'$$

(b)

$$②' \longleftarrow ③'$$

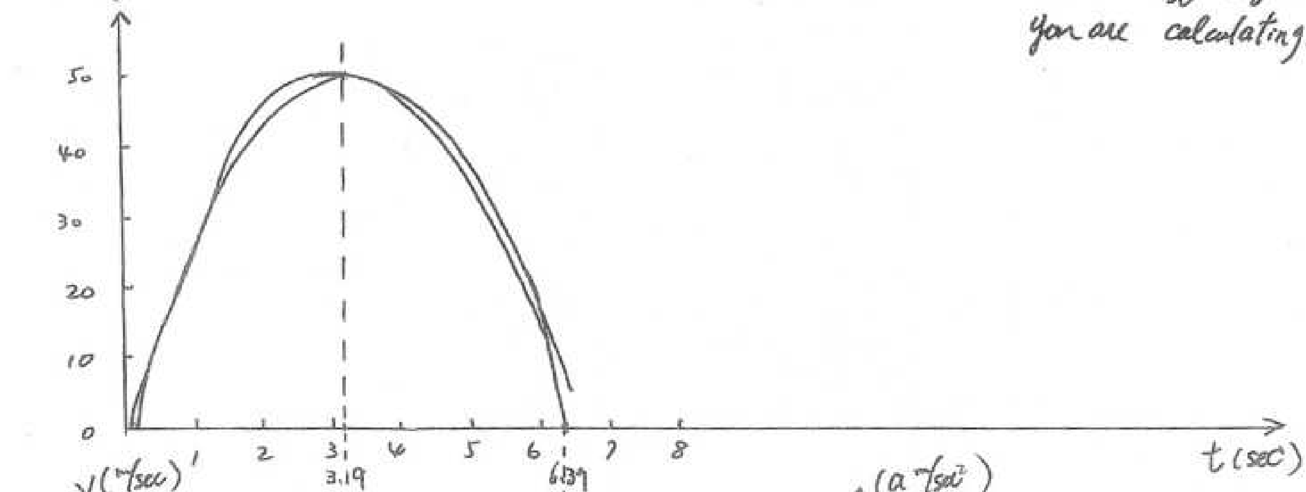
$$t = \frac{\sqrt{50 \cdot 2 \cdot g}}{g} = \sqrt{\frac{50 \cdot 2}{g}} = 3.19 \text{ sec}$$

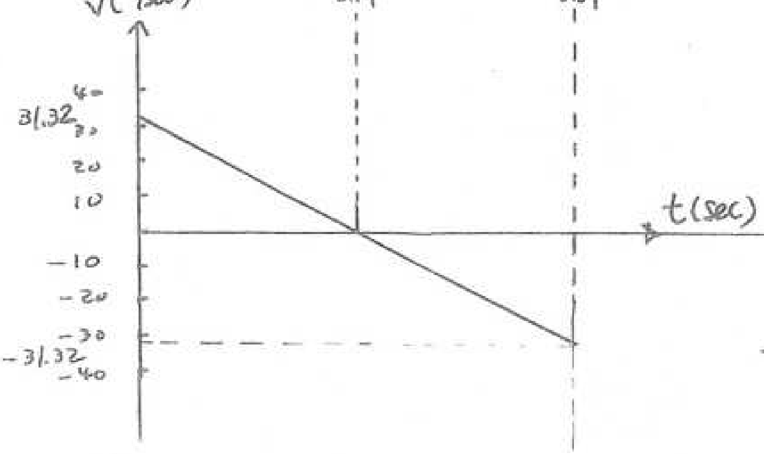
$$\text{time Total} = 3.19 \text{ sec} \overset{up}{\uparrow} + 3.19 \text{ sec} \underset{down}{\downarrow}$$

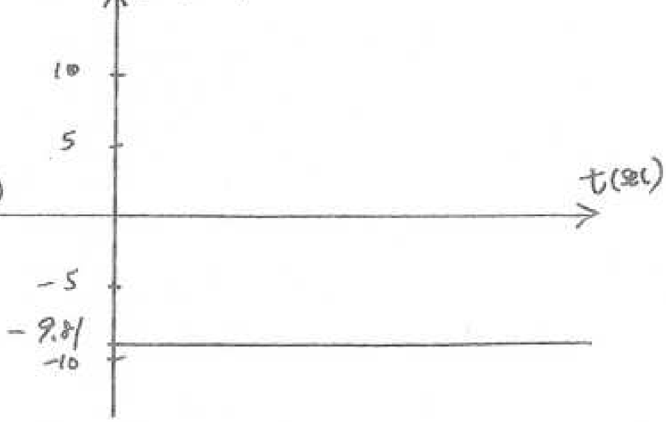
$$= \underline{\underline{6.39 \text{ sec} \quad (6.3855.. \text{ sec})}}$$

When you write your answer, you round off but do not use rounded off figures when you are calculating.

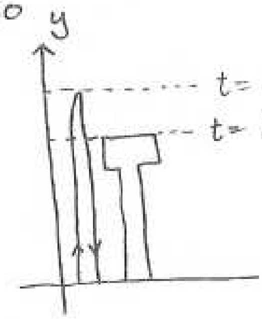
y (m)



# 50




( max height at 1.5 sec + 1.0 sec = 2.5 sec )

Once again start with basic eqns.

$$\begin{cases} a = -g & \text{——— ①} \\ V = \int a \cdot dt = \int -g \cdot dt = -gt + V_0 & \text{——— ②} \\ y = \int V \cdot dt = \int (-gt + V_0)\,dt = -\frac{1}{2}gt^2 + V_0 t + \cancel{y_0}^{0} & \text{— ③} \end{cases}$$

Eqn ②, $V = 0$ at $t = 2.5\text{sec}$

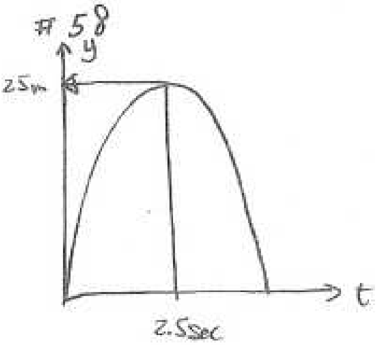
$0 = -gt + V_0$

$\therefore V_0 = 2.5g$ ——— ②'

③ ← ② @ $t = 1.5\text{sec}$ ( to reach the top of the tower )

$y = -\frac{1}{2}g(1.5)^2 + 2.5g(1.5) = \underline{\underline{25.75\,m \quad (25.75125\,m)}}$

# 58




From the graph, $y_{max}$ is 25m at $t = 2.5$ sec

Even this is on the other planet, physics is still the same!

$$\begin{cases} a = -g & \text{——— ①} \\ V = \int a \cdot dt = \int -g \cdot dt = -gt + V_0 & \text{——— ②} \\ y = \int V \cdot dt = \int -gt + V_0 = -\frac{1}{2}gt^2 + V_0 t + \cancel{y_0}^{0} & \text{——— ③} \end{cases}$$

(a) Eqn ② $y_{max}$ ($\equiv V = 0$ at $t = 2.5\text{sec}$)

$0 = -g(2.5) + V_0 \Rightarrow V_0 = 2.5g$ —— ②'

③ ← ②' ( $y = 25m$ )

$25 = -\frac{1}{2}g(2.5)^2 + (2.5g)(2.5)$

$= \frac{1}{2}g(2.5)^2$

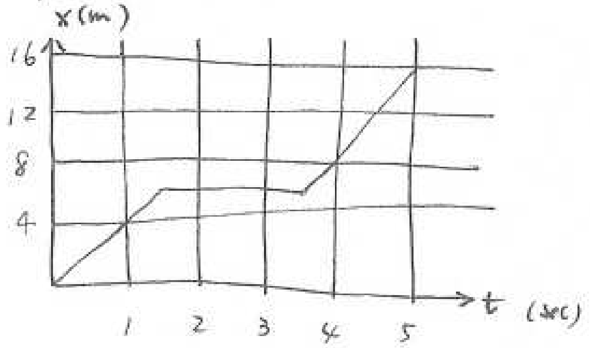
$\therefore g = \frac{2 \cdot 25}{(2.5)^2} = \underline{\underline{8.0\ m/sec^2}}$ ——— ③'

(b) ②' ← ③'

$V_0 = 2.5g$

$= 2.5\text{sec}(8.0\ m/sec^2) = \underline{\underline{20.0\ m/sec}}$

#78




(a) $x(t=0.5\text{sec}) = 2\text{m}$

$x(t=4.5\text{sec}) = 12\text{m}$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{12\text{m} - 2\text{m}}{4.5\text{sec} - 0.5\text{sec}} = \underline{\underline{2.5\ \text{m/sec}}}$$

(b) instantaneous vel $= \frac{dx}{dt}$ = slope of $x(t)$ fcn (as shown on the top)

Notice that the slope is const between $t = 4$ sec & 5 sec (because the line is straight)

$$\therefore \frac{dx}{dt} = \frac{\Delta x}{\Delta t} = \frac{x(t=5\text{sec}) - x(4\text{sec})}{5\text{sec} - 4\text{sec}}$$

$$= \frac{16\text{m} - 8\text{m}}{1\text{sec}} = \underline{\underline{8\ \text{m/sec}}}$$

(c)

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v(t=4.5\text{sec}) - v(t=0.5\text{sec})}{4.5\text{sec} - 0.5\text{sec}}$$

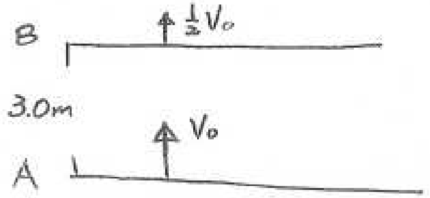
$$= \frac{8\ \text{m/sec} - 4\ \text{m/sec}}{4\text{sec}}$$

$v(t=0.5\text{sec})$ is calculated the same way used at $t=4.5$ sec in (b)

$$= \underline{\underline{1\ \text{m/sec}^2}}$$

(d) Since $v$ is const $a = \frac{dv}{dt} = \underline{\underline{0\ \text{m/sec}^2}}$

# 89.




let $V_0$ be the speed at A
Also, let A be the origin.

$$\begin{cases} a = -g & \text{—— ①} \\ V = \int a \cdot dt = -gt + V_0 & \text{—— ②} \\ y = \int v \cdot dt = \int(-gt + V_0)\,dt = -\frac{1}{2}gt^2 + V_0 t + \cancel{y_0}^{\,0} & \text{—— ③} \end{cases}$$

(a) at $y = 3.0\,m$, $V = \frac{1}{2}V_0$

Eqn. ②, solve for $t$

$$\frac{1}{2}V_0 = -gt + V_0$$

$$t = \frac{V_0}{2g} \quad \text{—— ②'}$$

③ ⟵ ②'

$$3 = -\frac{1}{2}g\left(\frac{V_0}{2g}\right)^2 + V_0\left(\frac{V_0}{2g}\right)$$

$$= -\frac{1}{2}\frac{V_0^2}{4g} + \frac{V_0^2}{2g}$$

$$= -\frac{V_0^2}{8g} + \frac{V_0^2}{2g} = \frac{3}{8}\frac{V_0^2}{g}$$

$$\therefore V_0 = \sqrt{3 \cdot \frac{8g}{3}} = \sqrt{8g} = \underline{\underline{8.859\ m/sec}}$$

(b) at the max. pt., $V = 0$.

Eqn ② Solve for $t$

$$0 = -gt + V_0$$

$$t = \frac{V_0}{g} \quad \text{—— ②'}$$

③ ⟵ ②'

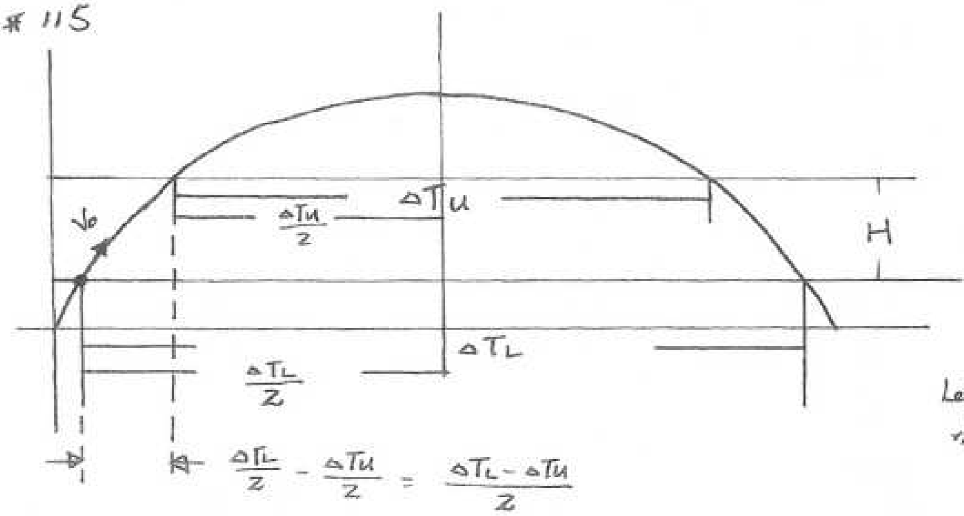
$$y = -\frac{1}{2}gt^2 + V_0 t$$

$$= -\frac{1}{2}g\left(\frac{V_0}{g}\right)^2 + V_0\left(\frac{V_0}{g}\right)$$

$$= \frac{1}{2}\frac{V_0^2}{g} = \frac{1}{2}\frac{(8.859)^2}{g} = 4\,m$$

$\therefore$ It is ($4m - 3m = \underline{\underline{1m}}$) higher than B

#115




Because there is only one constant acc. (of g), the left half & right half are symmetric to each other.
$\Rightarrow \frac{1}{2}$ total time to reach the max.

Let $V_0$ be the speed at the Lower level.

$$\begin{cases} a = -g & \text{①} \\ V = \int a \cdot dt = \int -g \cdot dt = -gt + V_0 & \text{②} \\ y = \int v \cdot dt = \int (-gt + V_0)\, dt = -\frac{1}{2} g t^2 + V_0 t + \cancel{y_0}^{\,0} & \text{③} \end{cases}$$

Eqn ② $\quad V = 0$ at $t = \frac{\Delta T_L}{2}$

$$0 = -g\left(\frac{\Delta T_L}{2}\right) + V_0$$

$$\therefore \quad V_0 = \frac{\Delta T_L\, g}{2} \quad \text{——— ③'}$$

③ ← ②' & $y = H$ at $t = \frac{\Delta T_L - \Delta T_U}{2}$

$$H = -\frac{1}{2} g \left(\frac{\Delta T_L - \Delta T_U}{2}\right)^2 + \frac{\Delta T_L \cdot g}{2}\left(\frac{\Delta T_L - \Delta T_U}{2}\right)$$

$$= -\frac{g}{8}(\Delta T_L - \Delta T_U)^2 + \frac{\Delta T_L\, g}{4}(\Delta T_L - \Delta T_U)$$

$$= \left(-\frac{1}{8}(\Delta T_L^2 - 2\Delta T_L \Delta T_U + \Delta T_U^2) + \frac{\Delta T_L^2 - \Delta T_L \Delta T_U}{4}\right) g$$

$$= \frac{1}{8}\left(-\Delta T_L^2 + 2\Delta T_L \cdot \Delta T_U - \Delta T_U^2 + 2\Delta T_L^2 - 2\Delta T_L \Delta T_U\right) g$$

$$= \frac{1}{8}\left(\Delta T_L^2 - \Delta T_U^2\right) g$$

$$\therefore \quad g = \frac{8H}{\Delta T_L^2 - \Delta T_U^2}$$