ch 3 #31, #37, #49

#31 Given:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = a_x b_x + a_y b_y + a_z b_z \quad \text{— (1)}$$

$$\vec{a} = 3.0\hat{i} + 3.0\hat{j} + 3.0\hat{k}$$

$$\vec{b} = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k}$$

$$\therefore |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$= \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2}$$

$$= \sqrt{27} \quad \text{— (2)}$$

$$|\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$$

$$= \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2}$$

$$= \sqrt{14} \quad \text{— (3)}$$

$$(1) \leftarrow (2), \& (3)$$

$$|\vec{a}||\vec{b}| \cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\sqrt{27}\sqrt{14} \cos\theta = (3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)$$

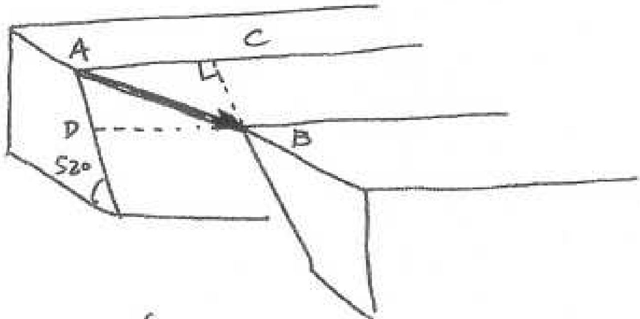
$$\therefore \cos\theta = \frac{18}{\sqrt{27}\sqrt{14}}$$

$$\therefore \theta = \cos^{-1}\frac{18}{\sqrt{27}\sqrt{14}}$$

$$= 22.2076543$$

$$\sim \underline{\underline{22.^{\circ}2}}$$

\# 37



pt. B is still touching the fault plane.

(a) If it slid from A to B

$\overline{AC} = 22m$

$\overline{AD} = 17m$

Hence $\vec{AB} = \vec{AC} + \vec{AD}$

and $|\vec{AB}| = \sqrt{(\vec{AC})^2 + (\vec{AD})^2}$

$= \sqrt{(22m)^2 + (17m)^2}$

$= \underline{\underline{27.8\,m}}$

(b)

If $|\vec{AD}| = 17m$ and $\measuredangle = 52°$

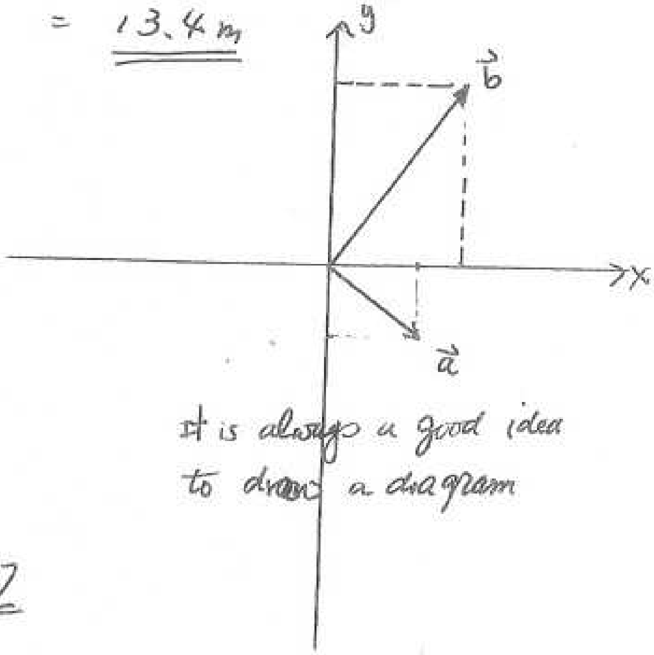
then $\overline{AF} = 17m \cdot \sin 52°$

$= \underline{\underline{13.4\,m}}$

\# 49.

$\vec{a} = (4.0m)\hat{i} - (3.0m)\hat{j}$

$\vec{b} = (6.0m)\hat{i} + (8.0m)\hat{j}$



It is always a good idea to draw a diagram

(a) $|\vec{a}| = \sqrt{a_i^2 + a_j^2}$

$= \sqrt{(4.0m)^2 + (-3.0m)^2}$

$= \underline{\underline{5m}}$

(b) $\alpha = \tan^{-1}\frac{a_j}{a_i} = \tan^{-1}\frac{-3}{4} = \underline{\underline{-36.87°}}$

(c) $|\vec{b}| = \sqrt{(6m)^2 + (8m)^2}$

$= \underline{\underline{10\ m}}$

(d) $\beta = \tan^{-1}\frac{8}{6} = \underline{\underline{53.^{\circ}13}}$

(e) $\vec{a} + \vec{b} = (4\hat{i} - 3\hat{j}) + (6\hat{i} + 8\hat{j})$

$= (4+6)\hat{i} + (-3+8)\hat{j}$

$= 10m\,i + 5m\hat{j}$

$|\vec{a} + \vec{b}| = \sqrt{(10m)^2 + (5m)^2}$

$= \underline{\underline{11.18m}}$

(f) $\gamma = \tan^{-1}\frac{5}{10} = \underline{\underline{26.^{\circ}57}}$

(g) $\vec{b} - \vec{a} = (6\hat{i} + 8\hat{j}) - (4\hat{i} - 3\hat{j})$

$= (6-4)\hat{i} + (8+3)\hat{j}$

$= \underline{\underline{2m\hat{i} + 11m\hat{j}}}$

(h) $\delta = \tan^{-1}\frac{11}{2} = \underline{\underline{79^{\circ}.70}}$

(i) $\vec{a} - \vec{b} = (4\hat{i} - 3\hat{j}) - (6\hat{i} + 8\hat{j})$

$= (4-6)\hat{i} + (-3-8)\hat{j}$

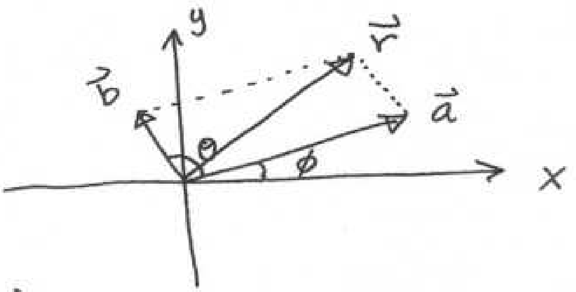
$= \underline{\underline{-2m\hat{i} - 11m\hat{j}}}$

(j) $\varepsilon = \tan^{-1}\frac{-11}{-2} = 259^{\circ}.7$ (be careful. Since x & y are both negative the vector is in the 3rd Quad.)

(k) As you can see they are opposite to each other.

$\rightarrow$ 180° apart.

Extra



$\vec{a} = a_x + a_y = |a| \cos\phi + |a| \sin\phi$

$\vec{b} = b_x + b_y = |b| \cos(\phi+\theta) + |b| \sin(\phi+\theta)$

$\vec{r} = \vec{a} + \vec{b} = (a_x + b_x) + (a_y + b_y)$

and

$|\vec{r}| = \sqrt{(a_x+b_x)^2 + (a_y+b_y)^2}$

$= \sqrt{(|a|\cos\phi + |b|\cos(\phi+\theta))^2 + (|a|\sin\phi + |b|\sin(\phi+\theta))^2}$

$= \sqrt{a^2\cos^2\phi + b^2\cos^2(\phi+\theta) + 2|a||b|\cos\phi\cos(\phi+\theta) + a^2\sin^2\phi + b^2\sin^2(\phi+\theta) + 2|a||b|\sin\phi\sin(\phi+\theta)}$

$= \sqrt{a^2(\underbrace{\cos^2\phi + \sin^2\phi}_{1}) + b^2(\underbrace{\cos^2(\phi+\theta) + \sin^2(\phi+\theta)}_{1}) + 2|a||b|(\cos\phi\cos(\phi+\theta) + \sin\phi\sin(\phi+\theta))}$

trig. Identity.

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

so

$\cos(a-b) = \cos a \cos(-b) - \sin a \sin(-b)$

$= \cos a \cos b + \sin a \sin b.$

$a = \phi$

$b = \phi + \theta.$

$= \sqrt{a^2 + b^2 + 2ab\cos(\phi - (\phi+\theta))}$

$= \sqrt{a^2 + b^2 + 2ab\cos(-\theta)} = \underline{\underline{\sqrt{a^2+b^2+2ab\cos\theta}}}$