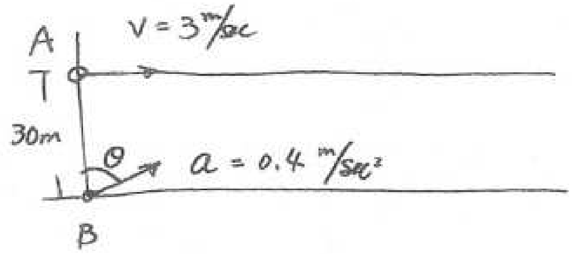
Ch. 4. #16, #19, #26, #85, #96

#16



A

x comp

$a_{Ax} = 0$ —— ①

$V_{Ax} = V = 3\ m/sec$ —— ②

$x_A = vt = 3t$ —— ③

y comp

$a_{Ay} = 0$ —— ④

$V_{Ay} = 0$ —— ⑤

$y_A = 30\ m$ —— ⑥

B

x comp

$a_{Bx} = a \sin\theta$ —— ⑦

$V_{Bx} = at \sin\theta$ —— ⑧

$x_B = \frac{1}{2}at^2 \sin\theta$ —— ⑨

y comp

$a_{By} = a \cos\theta$ —— ⑩

$V_{By} = at \cos\theta$ —— ⑪

$y_B = \frac{1}{2}at^2 \cos\theta$ —— ⑫

At collision

$x_A = x_B$ (③=⑨) & $y_A = y_B$ (⑥=⑫)

$3t = \frac{1}{2}at^2 \sin\theta$

$30 = \frac{1}{2}at^2 \cos\theta$ — ⑥′

$3 = \frac{1}{2}at \sin\theta$

$\therefore t = \frac{6}{a \sin\theta}$ — ③′

⑥′ ⟵ ③′

$30 = \frac{1}{2}a \left(\frac{6}{a\sin\theta}\right)^2 \cos\theta$

$60 = \frac{6^2}{a \sin^2\theta} \cos\theta$

$5a \sin^2\theta = 3\cos\theta$ (since $a = 0.4\,m/sec^2$)

$5a = 2\,m/sec^2$

$2(1-\cos^2\theta) - 3\cos\theta = 0$

$2\cos^2\theta + 3\cos\theta - 2 = 0.$

$$\cos\theta = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2\cdot 2}$$

$$= \frac{2}{4} \text{ or } \cancel{-2} \text{ not possible}$$

$$\therefore \theta = \cos^{-1}\tfrac{1}{2} = \underline{\underline{60°}}$$

#19

x comp

$a_x = 0$ ——— ①

$V_x = \int a_x\,dt = V_{ox}$ — ②

$X = \int V_x\,dt = V_{ox}\,t$ — ③

y comp

$a_y = -g$ ——— ④

$V_y = \int a_y\cdot dt = -gt + V_{oy}$ — ⑤

$y = \int V_y\cdot dt = -\frac{1}{2}gt^2 + V_{oy} + y_o$ — ⑥

at $y = 9.1m$, $V_x = 7.6\,m/sec$ & $V_y = 6.1\,m/sec$

there are two different ways to put the origin. Since it asks the total horizontal distance, I will use the ground as the origin (then, $y_o = 0m$)

Eqn. ⑤, $V_y = 6.1\,m/sec$. Solve for t

$$6.1 = -gt + V_{oy}$$

$$t = \frac{V_{oy} - 6.1}{g} \text{ ——— ⑤'}$$

⑥ ⟵ ⑤' ($y = 9.1m$ at that moment)

$$9.1 = -\tfrac{1}{2}gt^2 + V_{oy}t$$

$$= -\tfrac{1}{2}g\left(\frac{V_{oy}-6.1}{g}\right)^2 + V_{oy}\left(\frac{V_{oy}-6.1}{g}\right)$$

$$= \frac{V_{oy}^2 - 6.1^2}{2g}$$

$$\therefore\ V_{oy} = \sqrt{(9.1)(2)(g) + 6.1^2}$$

$$= 14.69 \quad (14.6884989)\ m/sec \quad \text{—— ⑥}'$$

(a) At the top, $V = 0\ m/sec$

Eqn. ⑤. Solve for t

$$0 = -gt + V_{oy}$$

$$t = \frac{V_{oy}}{g} \quad \text{—— ⑥}''$$

$$⑥ \leftarrow ⑤''$$

$$y = -\frac{1}{2}gt^2 + V_{oy}t$$

$$= -\frac{1}{2}g\left(\frac{V_{oy}}{g}\right)^2 + V_{oy}\left(\frac{V_{oy}}{g}\right)$$

$$= -\frac{1}{2}g\frac{V_{oy}^2}{g^2} + \frac{V_{oy}^2}{g}$$

$$= \frac{1}{2}\frac{V_{oy}^2}{g} = \underline{\underline{10.9965\,m}}$$

(b) Eqn. ⑥. $y = 0$. Solve for t

$$0 = -\frac{1}{2}gt^2 + V_{oy}t$$

$$t\left(-\frac{1}{2}gt + V_{oy}\right) = 0$$

$$-\frac{1}{2}gt + V_{oy} = 0$$

$$t = \frac{2V_{oy}}{g} = 2.994597126\ sec \quad \text{—— ⑥}'$$

$$③ \leftarrow ⑥'$$

$$x = V_{ox}t$$

$$= 7.6 \times 2.99\ldots$$

$$= \underline{\underline{22.76\,m}}$$

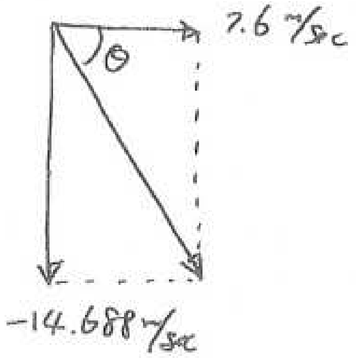
(c) $② \,\&\, ⑤ \leftarrow ⑥'$

$V_x = 7.6\ m/sec$ (No acc. toward x direction)

$$V_y = -gt + V_{oy} = -14.6884989\ m/sec$$

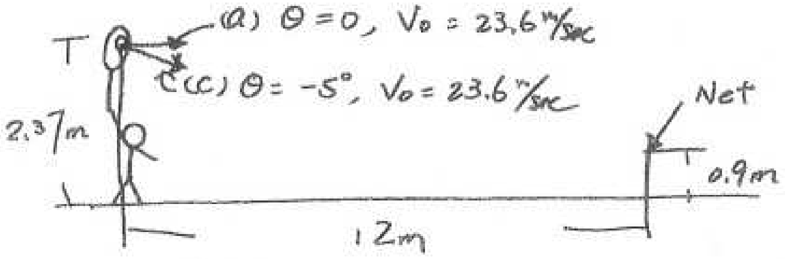
$$|V| = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{(7.6)^2 + (-14.688)^2} = \underline{\underline{16.538 \text{ m/sec}}}$$



$$\theta = \tan^{-1}\frac{V_y}{V_x} = \underline{\underline{-62.°64}}$$

#26.



(a) $\theta = 0°$

$\underline{x \text{ comp}}$

$a_x = 0$ ——— ①

$V_x = V_0 = 23.6$ m/sec ——— ②

$x = V_0 t$ ——— ③

$\underline{y \text{ comp}}$

$a_y = -g$ ——— ④

$V_y = -gt + \cancel{V_{0y}}^{\,0}$ ——— ⑤

$y = -\frac{1}{2}gt^2 + \cancel{y_0}^{\,2.37}$ ——— ⑥

Eqn. ③, $x = 12$ m, solve for t

$12 = V_0 t$

$t = \frac{12}{V_0}$ ——— ③'

⑥ ⟵ ③'

$y = -\frac{1}{2}gt^2 + 2.37$

$= -\frac{1}{2}g\left(\frac{12}{V_0}\right)^2 + 2.37$

$= -1.26817 + 2.37$

$= 1.10182993$ m ... $\underline{\text{yes, it does clear the net}}$

(b) $(1.1018 - 0.9)$ m $= 0.2018$ m (~ 20 cm above the net → Bad serve!)

(c) $\theta = -5°$

<u>x comp</u>

$a_x = 0$ ——— ①

$V_x = V_0 \cos(-5°) = V_0 \cos 5°$ ——— ②

$X = V_0 t \cos 5°$ ——— ③

<u>y comp</u>

$a_y = -g$ ——— ④

$V_y = -gt + V_0 \sin(-5°)$

$= -gt - V_0 \sin 5°$ ——— ⑤

$y = -\frac{1}{2}gt^2 + V_0 t \sin 5° + 2.37$ ——— ⑥

Eqn ③, $x = 12\,m$, solve for t

$12 = V_0 t \cos 5°$

$t = \frac{12}{V_0 \cos 5°}$ ——— ③'

⑥ ← ③'

$$y = -\frac{1}{2} g \left(\frac{12}{V_0 \cos 5°}\right)^2 - \cancel{V_0}\left(\frac{12}{\cancel{V_0}\cos 5°}\right)\sin 5° + 2.37$$

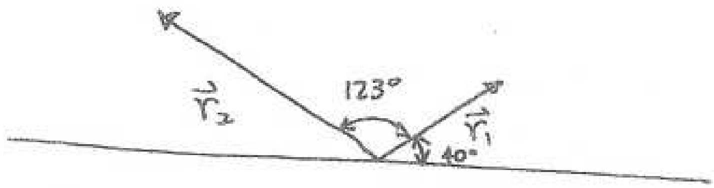
$$= -\frac{1}{2} g \frac{12^2}{(23.6 \cos 5°)^2} - 12 \tan 5° + 2.37$$

$$= -1.277876977 - 1.0499 + 2.37 = -2.3277694\,m + 2.37 = 0.0423\,m$$

⇒ No it does not clear the net.

$0.9 - 0.0423 = 0.8577\,m$ (~86 cm below the net → Fault!)

#85.



$\vec{r}_1 = 360\,m, 40°$

$\vec{r}_2 = 790\,m, (40° + 123°)$

$\vec{r}_1 = |\vec{r}_1| \cos 40°\, \hat{i} + |\vec{r}_1| \sin 40°\, \hat{j}$

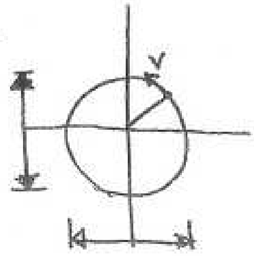
$\vec{r}_2 = |\vec{r}_2| \cos(40° + 123°)\, \hat{i} + |\vec{r}_2| \sin(40° + 123°)\, \hat{j}$

$$\text{Displacement} = \sqrt{(r_{2x} - r_{1x})^2 + (r_{2y} - r_{1y})^2}$$

$$= \sqrt{((790)\cos 163° - (360)\cos 40°)^2 + ((790)\sin 163° - (360)\sin 40°)^2}$$

$$= \underline{\underline{1031.26\,m}}$$

#96.



Basic circular eqn.

a circular motion is a combination of X motion – $A \cos(\omega t + \phi)$ & y motion – $A \sin(\omega t + \phi)$, where A is an amplitude, ω is an angular speed, and ϕ is a phase shift. If you are not sure of these, you might want to brush up "Trig."

In this case, $r = 3m \Rightarrow A = 3m$

Period = 20 sec → 2π rad takes 20 sec → $\omega = \frac{2\pi \text{ rad}}{20 \text{sec}} = \frac{\pi}{10}$ rad/sec

$X_{origin} = 0$

y_{origin} = 3m below the basic eqn.

$\rightarrow y = 3 + A \sin(\omega t + \phi)$

at t = 0 sec, the particle is at (0,0) → $\frac{\pi}{2}$ rad behind the basic eqn. → $\phi = -\frac{\pi}{2}$

So

$$\begin{cases} x = 3 \cos\left(\frac{\pi}{10} t - \frac{\pi}{2}\right) \\ y = 3 + 3 \sin\left(\frac{\pi}{10} t - \frac{\pi}{2}\right) \end{cases}$$

(a) $X(t = 5\text{sec}) = 3\cos\left(\frac{\pi}{10}(5) - \frac{\pi}{2}\right) = 3m$

$y(t = 5\text{sec}) = 3 + 3\sin\left(\frac{\pi}{10}(5) - \frac{\pi}{2}\right) = 3m$ (3, 3)

$\vec{r} = 3_m \hat{i} + 3_m \hat{j}$

(b) $x(t = 7.5\text{sec}) = 2.121$ m

$y(t = 7.5\text{sec}) = 5.121$ m

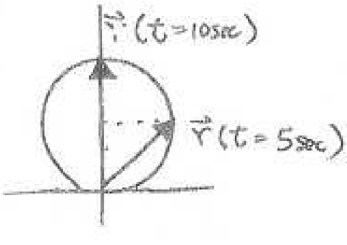
$\vec{r} = 2.121\text{m } \hat{i} + 5.121 \text{ m } \hat{j}$

(c) $X(t = 10\text{sec}) = 0$ m

$y(t = 10\text{sec}) = 6$ m

$\vec{r} = 0_m \hat{i} + 6 m \hat{j}$

(d)

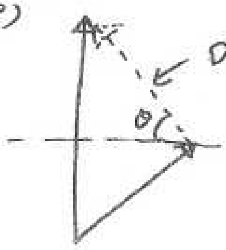


Displacement $= \sqrt{\Delta X^2 + \Delta y^2}$

$= \sqrt{(X_f - X_i)^2 + (y_f - y_i)^2}$

$= \sqrt{(0-3m)^2+(6m-3m)^2} = \sqrt{9+9}\,m = \underline{\underline{3\sqrt{2}\,m}}$

(e)



As you can see, since $\Delta x = -3m$ & $\Delta y = 3m$, $\underline{\underline{\theta = 45°}}$. speed $= \frac{\Delta D}{\Delta t} = \frac{3\sqrt{2}\,m}{5} = 0.6\sqrt{2}\ m/sec$

$= \underline{\underline{0.8485\ m/sec}}$

You can do this by calculating V_x & V_y separately.

(f) & (g)

$V_x = \frac{dx}{dt} = -3\frac{\pi}{10}\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right)$ $\quad V_y = \frac{dy}{dt} = 3\frac{\pi}{10}\cos\left(\frac{\pi}{10}t - \frac{\pi}{2}\right)$

$$\left.\begin{array}{l} V_x(t=5sec) = 0\ m/sec \\ V_y(t=5sec) = \frac{3\pi}{10}\ m/sec \end{array}\right\} \underline{\underline{V = 0\ m/sec\ \hat{i} + \frac{3\pi}{10}\ m/sec\ \hat{j}}}$$

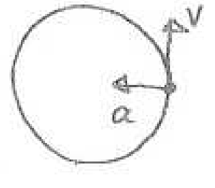
$$\left.\begin{array}{l} V_x(t=10sec) = -\frac{3\pi}{10}\ m/sec \\ V_y(t=10sec) = 0\ m/sec \end{array}\right\} \underline{\underline{V = -\frac{3\pi}{10}\ m/sec\ \hat{i} + 0\hat{j}}}$$

$$\left\{\begin{array}{l} a_x = \frac{dV_x}{dt} = -3\frac{\pi^2}{100}\cos\left(\frac{\pi}{10}t - \frac{\pi}{2}\right) \\ a_y = \frac{dV_y}{dt} = -3\frac{\pi^2}{100}\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right) \end{array}\right.$$

$$\left.\begin{array}{l} a_x(t=5sec) = -\frac{3\pi^2}{100}\ m/sec^2 \\ a_y(t=5sec) = 0\ m/sec^2 \end{array}\right\} \underline{\underline{a = -\frac{3\pi^2}{100}\ m/sec^2\ \hat{i} + 0\ m/sec^2\ \hat{j}}}$$

$$\left.\begin{array}{l} a_x(t=10sec) = 0\ m/sec^2 \\ a_y(t=10sec) = -\frac{3\pi^2}{100}\ m/sec^3 \end{array}\right\} \underline{\underline{a = 0\ m/sec^2\ \hat{i} - \frac{3\pi^2}{100}\ m/sec^2\ \hat{j}}}$$

These results are expected because at t = 5sec. the particle is at (3,3). V is always tangent. So V at that point has to have only positive y and its acc. should have only negative x.



At t = 10 sec, the particle is at (0,6). V should be negative x direction only and a should be negative y only.