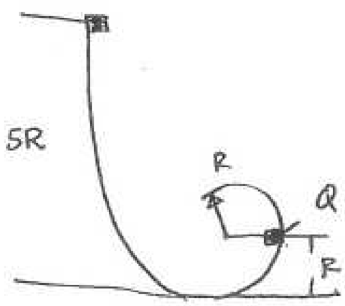
Ch. 8 #7, 8, 23, 29, 70, 85, 135

#7



(a) work done by gravitational force $= mg\Delta h$

$\therefore mg(5R - R)$

$= \underline{\underline{4mgR}}$

b) $mg(5R - 2R)$

$= \underline{\underline{3mgR}}$

(c) $\underline{\underline{5mgR}}$

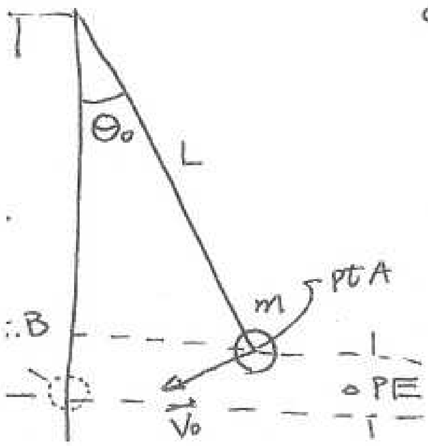
(d) $\underline{\underline{mgR}}$

(e) $\underline{\underline{2mgR}}$

(f) the same since we are talking ~~about~~ gravitational Pot. energy.

Now. Can you answer the net force the block feels at Q and at the top of the loop?

8.



(a) Lost of ΔPE is Gain of ΔKE.

At Pt. A.

$E_{Total} = PE + KE$

$= mg(L - L\cos\theta_0) + \frac{1}{2}mV_0^2$

$= mgL(1 - \cos\theta_0) + \frac{1}{2}mV_0^2$

At B

$E_{Total} = KE$

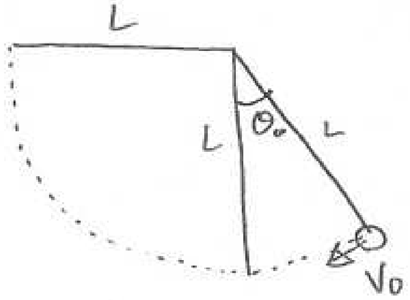
$= \frac{1}{2}mV^2$

Cons. of Energy

$$mgL(1-\cos\theta_0) + \frac{1}{2}mV_0^2 = \frac{1}{2}mV^2$$

$$V = \pm\sqrt{2gL(1-\cos\theta_0) + V_0^2}$$ (± shows directions)

b)



$$E_i = E_f$$

$$\frac{1}{2} m V_0^2 + mgL(1-\cos\theta_0) = mgL$$

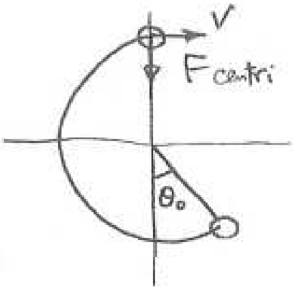
$$\frac{1}{2} V_0^2 = gL - gL(1-\cos\theta_0)$$

$$V_0^2 = 2(gL - gL + gL\cos\theta_0)$$

$$= 2gL\cos\theta_0$$

$$\therefore \underline{\underline{V = \pm\sqrt{2gL\cos\theta_0}}}$$

c)



At the top of its motion:
Because the string is straight, we need a centripetal force. The minimum centripetal force needed is its own weight!

$$F_{centri} = m\frac{V^2}{r} = mg$$

$$\Downarrow$$

$$\frac{V^2}{L} = g$$

$$\therefore V = \sqrt{gL}$$ (So, at the top, the block moves at $V = \sqrt{gL}$)

$$E_{total\ at\ Top} = PE + KE$$

$$= mg(2L) + \frac{1}{2}mV^2$$

$$= 2mgL + \frac{1}{2}m(gL)$$

$$= \frac{5}{2}mgL$$

$$E_i = E_f$$

$$\frac{1}{2}mV_0^2 + mgL(1-\cos\theta_0) = \frac{5}{2}mgL$$

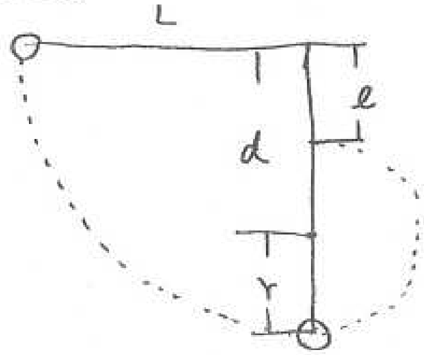
$$V_0^2 = 5gL - 2gL + 2gL\cos\theta_0$$

$$= 3gL + 2gL\cos\theta_0$$

$$\underline{\underline{V = \sqrt{gL(3+2\cos\theta_0)}}}$$

(d) If θ_0 is greater, the original PE is greater.
Hence the required V_0 is smaller for (b) & (c)

#23



$$E_i = E_f$$

$$PE = KE$$

(a) $$mgL = \frac{1}{2} m V^2$$

$$V = \sqrt{2gL}$$

$$= \underline{\underline{4.8522 \text{ m/sec}}}$$

(b) $d = 0.75\,m$ $\quad \therefore r = (1.2 - 0.75)\,m$
$= 0.45\,m$

Hence at the top of the smaller circular motion

$$E_i = E_f$$

$$PE_i = PE_f + KE$$

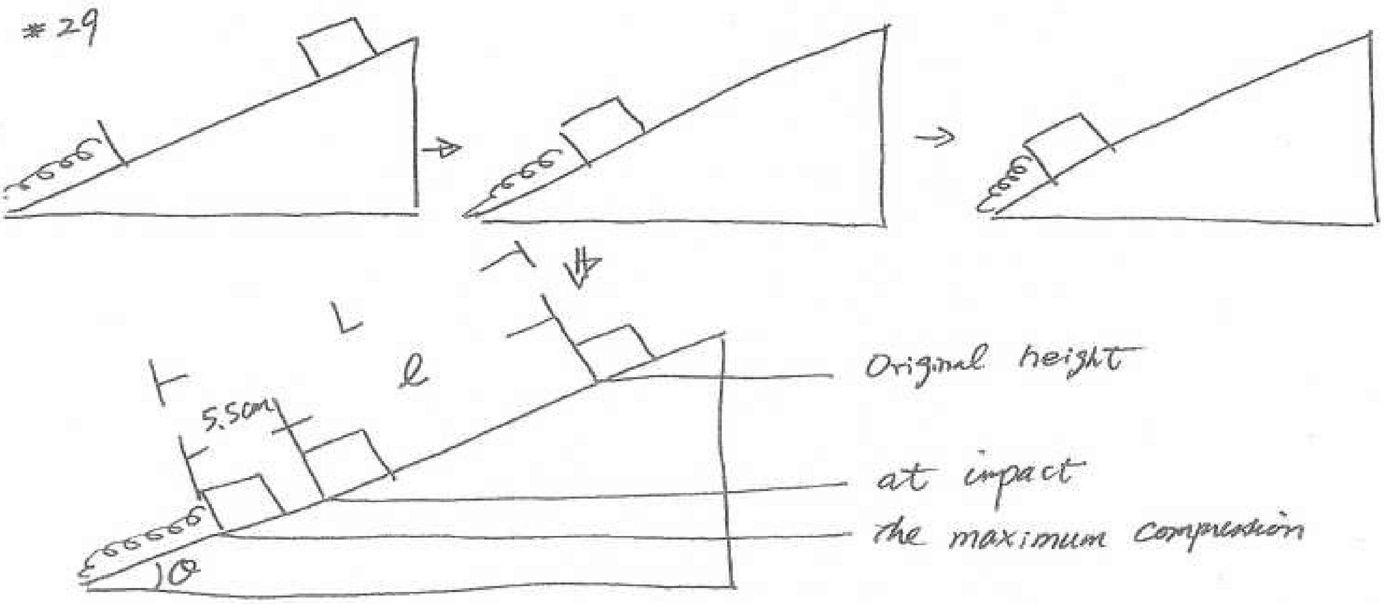
$$mgL = mg(2r) + \frac{1}{2} m V^2$$

$$\frac{1}{2} m V^2 = mg(L - 2r)$$

$$V = \sqrt{2g(L-2r)}$$

$$= \underline{\underline{2.426 \text{ m/sec}}}$$

#29



the easiest way to think about this problem is that the original P.E. was converted into S.E. at the max. compression. 8-4

(a) $k = \frac{270N}{0.02m} = 13500\ N/m$

$m = 12\,kg$

$\theta = 30^\circ$

$E_i = E_f$

$mgh = \frac{1}{2}kx^2$

$mgL\sin 30^\circ = \frac{1}{2}(13500\ N/m)(0.055)^2$

$mg(l + 0.055)\sin 30^\circ$

Solve for l

$l = 0.29190366 9\,m$

So

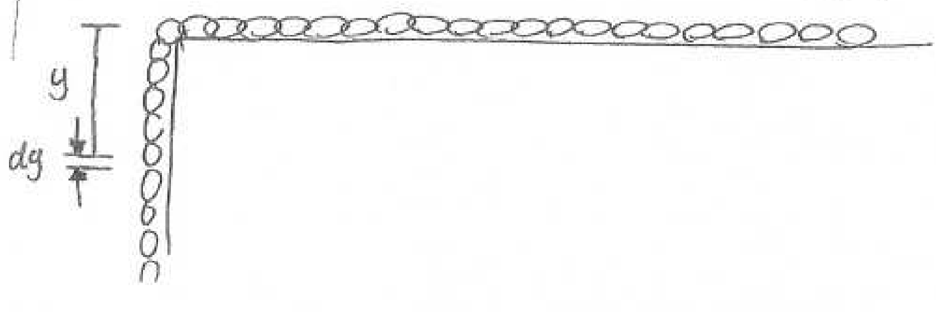
$L = l + 0.055 = \underline{\underline{0.3469\,m}}$

(b) $mgl\sin 30^\circ = \frac{1}{2}mV^2$

$V = \sqrt{2gl\sin 30^\circ} = \underline{\underline{1.6922\ m/sec}}$

#70 Do not think about the <u>whole</u> hanging part. Just like other times I have been stressing, think about a point (In this case, a small piece of the hanging chain). This little piece is "y" distance away from the top and its length is "dy" with its mass "dm". ("d" for anything small.) the relationship between length & mass is:

$dm = \lambda \cdot dy$ where "λ" is a (linear) density of the chain, $\frac{m}{L}$.



the work required to bring "dm" from where it is to the top of the table is

$$dw = (dm)\,g\,y \qquad (\text{just like } w = mgh)$$

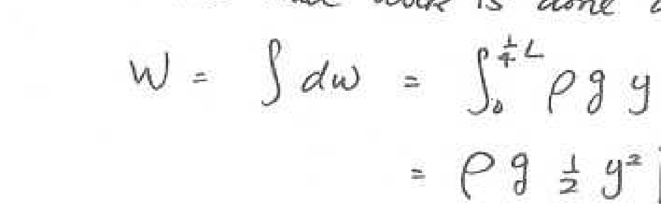
So,

$$dw = (\rho\,dy)\,g\,y \qquad (\text{changing from two variables to one variable})$$
$$= \rho g\, y.dy$$

So the total work is done when y changes from 0 to $\frac{1}{4}L$

$$W = \int dw = \int_0^{\frac{1}{4}L} \rho g\, y\,.dy$$
$$= \rho g \tfrac{1}{2} y^2 \Big|_0^{\frac{1}{4}L}$$
$$= \rho g \tfrac{1}{2} (\tfrac{1}{4}L)^2 = \tfrac{1}{32} \rho g L^2$$
$$= \tfrac{1}{32} \tfrac{m}{L} g L^2 = \underline{\underline{\tfrac{1}{32} mgL}}$$

Being able to think this way is very very important. Later on this semester (and in 231), we have to be able to use this method. (that is why you are taking Calculus). the following way is a solution using pre-calculus.



The total mass of the hanging part $\rightarrow \frac{1}{4}m$

the dist. from the top to the c.m. of the hanging part $\rightarrow \frac{1}{8}L$

$$W = \Delta PE = mg\Delta h = (\tfrac{1}{4}m)\cdot g\,(\tfrac{1}{8}L)$$
$$= \underline{\underline{\tfrac{1}{32} mgL}}$$

This looks easier. However, its almost impossible to solve this type of question with this method if the density of the chain is not constant.

However with calculus, even if the density is not const, as long as the density fcn. is given. we can solve it.

\# 85 $P = 1.5\,MW = 1.5 \times 10^6\,w$

$V_i = 10\,m/sec$ (at $t_i = 0\,sec$)

$V_f = 25\,m/sec$ (at $t_f = 360\,sec$)

This is a greate problem to check your understanding in:

- Force & Work relation
- Work - Energy theorem
- Work - Power relation

You should be able to explain them in writing (Not eqns only)

Also, students usually assume the force is const., but the problem does not state so. It is safe to assume it is not so if it is not stated so. However, P is const. in this case.

(a)

$$W = \int F \cdot dx = \int P \cdot dt$$

$$\int ma \cdot dx = \bar{P} \cdot t \quad (\text{P is const. in this case})$$

$$\int m \frac{dv}{dt} \cdot dx = \bar{P} \cdot t$$

$$\int m \frac{dx}{dt} \cdot dv = \bar{P} \cdot t$$

$$\int m\, v \cdot dv = \bar{P} \cdot t$$

$$\frac{1}{2} m v^2 \Big|_{V_i}^{V_f} = \bar{P} \cdot t$$

$$\therefore \bar{P} \cdot t_f = \frac{1}{2} m \left(V_f^2 - V_i^2 \right) \quad \text{—— ①}$$

$$\therefore m = \frac{2 \bar{P} t_f}{V_f^2 - V_i^2} = \frac{2 \cdot 1.5 \times 10^6\,w \cdot 360\,sec}{(25\,m/sec)^2 - (10\,m/sec)^2}$$

$$= 2.057 \times 10^6\,kg \quad (2057142.857\,kg)$$

(b) Instead of using t_f in eqn. ①, we can use $0 \le t \le 6\,min$

$$\therefore \bar{P} t = \frac{1}{2} m \left(V^2 - V_i^2 \right)$$

$$\therefore V = \sqrt{\frac{2 \bar{P} t}{m} + V_i^2} \quad \text{—— ②}$$

$$= \sqrt{\frac{2 \bar{P} t}{\frac{2 \bar{P} t_f}{V_f^2 - V_i^2}} + V_i^2}$$

$$= \sqrt{(V_f^2 - V_i^2)\frac{t}{t_f} + V_i^2}$$

$$= \sqrt{\frac{((25\,m/sec)^2 - (10\,m/sec)^2)}{360\,sec}\cdot t + (10\,m/sec)^2}$$

$$= \underline{\underline{\sqrt{1.458\dot{3}\,t + 100}\ \ m/sec}}$$

(c) Using eqn ②.

$$a = \frac{dv}{dt} = \frac{1}{2}\left(\frac{2\bar{P}t}{m} + v_i^2\right)^{-\frac{1}{2}}\frac{2P}{m}$$

$$= \frac{P}{m}\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{-\frac{1}{2}}$$

$$\therefore F = ma = P\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{-\frac{1}{2}}$$

$$= \underline{\underline{\frac{1.5\times 10^6}{\sqrt{1.458\dot{3}t + 100}}\ N}}$$

(d)
$$X = \int_0^{360} v\cdot dt$$

$$= \int\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{1/2}\cdot dt$$

$$= \frac{m}{2\bar{P}}\cdot\frac{2}{3}\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{3/2}\Big|_0^{360}$$

$$= \frac{m}{3\bar{P}}\left(\frac{2\bar{P}}{m}t + V_i^2\right)^{3/2}\Big|_0^{360}$$

$$= \frac{2\bar{P}t_f}{3\bar{P}(V_f^2 - V_i^2)}\left(1.458\dot{3}t + 100\right)^{3/2}\Big|_0^{360}$$

$$= \frac{2\cdot 360}{3(25^2 - 10^2)}\left[(1.458\dot{3}\cdot 360 + 100)^{3/2} - (100)\right]$$

$$= 6.687771523\times 10^3\,m$$

$$= \underline{\underline{6.69\times 10^3\,m}}$$

Work - Energy theorem.

A positive work done to a system causes the system to gain K.E. (a negative work can be done as well)

$$W = \Delta K.E.$$

Force - Work Relation

To change KE ($= \frac{1}{2} m V^2$), acceleration is needed which, of course, is caused by an outside force. Hence, outside force causes work. (Positive Force onto the system causes positive work done to the system) That's why this is the dot product.

$$W = \int \vec{F} \cdot d\vec{x} = \frac{1}{2} m (V_f^2 - V_i^2) = \Delta KE$$

Work - Power Relation

Power is a rate of change of work done w.R.T. time

$$P = \frac{dw}{dt}$$

Over all

$W = \Delta KE$ can be calculated by two ways

$$W = \int \vec{F} \cdot d\vec{x} = \int P \cdot dt$$

A typical mistake students make is that they mix the variables. Integrate $\underline{F}$ w.R.T. <u>distance</u> and $\underline{P}$ w.R.T. <u>time</u>. You can see if you don't practice, you can be easily confused. - this is the difference between Math & Physics. Physics does not tell you integrate what w.R.T. what. Make sure you know the relations.

\# 135

(a)
$$W = \int \vec{F}(r) \cdot d\vec{r}$$
$$= -\int_{\infty}^{R} G \frac{m_1 m_2}{r^2} \cdot d\vec{r}$$
$$= G m_1 m_2 \left. \frac{1}{r} \right|_{\infty}^{R}$$
$$= G m_1 m_2 \left(\frac{1}{R} - \frac{1}{\infty} \right)$$
$$= \frac{G m_1 m_2}{R}$$

(I hate to use X in space. there is no horizontal direction. Also, forces are the same if the distances are the same for any direction)

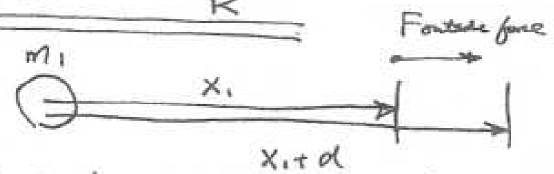
$\vec{F} \cdot \vec{r} = |\vec{F}||\vec{r}| \cos 180°$ (they are opposite)
$= -|\vec{F}||\vec{r}|$

$PE(r \to \infty) = 0$

This shows that the gravitational work did + work onto m_2 if brought ∞ to R. $\Delta KE = -\Delta PE$

$$\therefore \; PE = -\frac{G m_1 m_2}{R}$$

(b)



Outside force to bring m_2 from x_1 to $x_1 + d$ does a positive work (Not Gravitational Force)

(Gain of energy)

$$W = \int \vec{F}(r) \cdot d\vec{r}$$

this case $\vec{F}$ & $\vec{r}$ are the same direction.

$$= \int_{x_1}^{x_1+d} G \frac{m_1 m_2}{r^2} \cdot dr$$
$$= G m_1 m_2 \left. -\frac{1}{r} \right|_{x_1}^{x_1+d}$$
$$= G m_1 m_2 \left(\frac{1}{x_1} - \frac{1}{x_1 + d} \right)$$
$$= G m_1 m_2 \left(\frac{(x_1+d) - x_1}{x_1 (x_1 + d)} \right)$$
$$= G m_1 m_2 \frac{d}{x_1 (x_1 + d)}$$