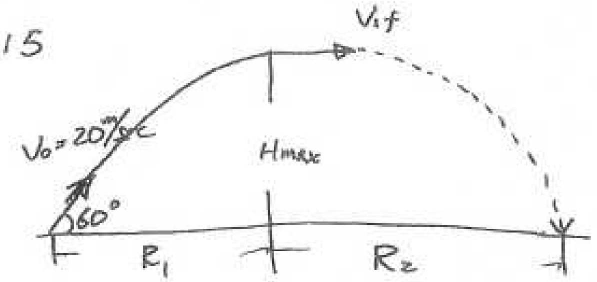
ch. 9. #15, 17, 20, 27, 49, 85, 112, 130

#15



Eqns. until the shell reaches its max (H_{max})

x		y	
$a_x = 0$	①	$a_y = -g$	④
$V_x = V_0 \cos 60°$	②	$V_y = -gt + V_0 \sin 60°$	⑤
$X = V_0 t \cos 60°$	③	$y = -\frac{1}{2} g t^2 + V_0 t \sin 60°$	⑥

For H_{max}.

Eqn ⑤ $V_y = 0$ solve for t

$$0 = -gt + V_0 \sin 60°$$

$$gt = V_0 \sin 60°$$

$$t = \frac{V_0 \sin 60°}{g} \quad \text{—— ⑤}'$$

⑥ ← ⑤'

$$y = -\frac{1}{2} g \left(\frac{V_0 \sin 60°}{g}\right)^2 + V_0 \left(\frac{V_0 \sin 60°}{g}\right) \sin 60°$$

$$= \frac{1}{2}\left(\frac{V_0^2 \sin^2 60°}{g}\right) \equiv H_{max} = 15.29051988\,m$$

③ ← ⑤'

$$X = V_0 \left(\frac{V_0 \sin 60°}{g}\right) \cos 60° \equiv R_1 = 17.65597153\,m$$

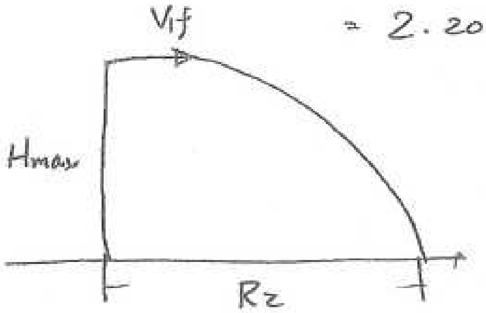
Eqns. after it breaks into two parts

$$\begin{array}{ccc} p_{ix} & & p_f \\ m_i V_{ix} & = & m_1 V_{1f} + m_2 \cancel{V_{2f}}^{\,0} \\ \Downarrow & m_1 = m_2 = \frac{1}{2} m_i \equiv m & \Downarrow \\ 2m V_{ix} & = & m V_{1f} \end{array}$$

$$\therefore \quad V_{1f} = 2 V_{ix}$$

$$V_{1f} = 2V_0 \cos 60^\circ$$

$$= 2 \cdot 20 \cdot \tfrac{1}{2} = 20 \ \text{m/sec}$$



x

$a_x = 0$ — ①

$V_x = 20 \ \text{m/sec}$ — ②

$X = 20t$ — ③

y

$a_y = -g$ — ④

$V_y = -gt + \cancel{V_{0y}}^{\,0}$ — ⑤

$y = -\frac{1}{2}gt^2 + \cancel{y_0}^{\,H_{max}}$ — ⑥

to calculate R_2:

Eqn. ⑥ $y=0$, solve for t

$$0 = -\tfrac{1}{2}gt^2 + H_{max}$$

$$\tfrac{1}{2}gt^2 = H_{max}$$

$$t = \sqrt{\frac{2H_{max}}{g}} \quad \text{— ⑥}'$$

③ ← ⑥'

$$X = 20\sqrt{\frac{2H_{max}}{g}} = 35.31194307 \, m$$

Range $= R_1 + R_2 = 52.97 \, m$ $(52.96791460 \, m)$

This ans. is exactly $3R_1$ meaning $R_2 = 2R_1$.
Can you justify why? (You should be able to ... it is an important conceptual question.)

Also, if this event happend on another place,

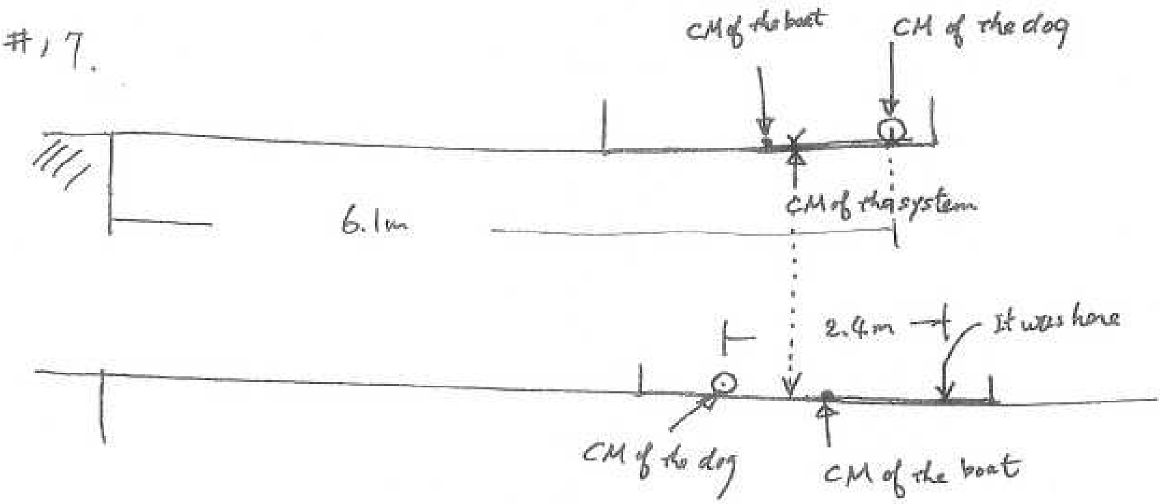
$$R_1 = \frac{2V_0^2 \sin 60^\circ \cos 60^\circ}{a}$$

$$R_2 = \frac{2V_0^2 \sin 60^\circ \cos 60^\circ}{a}$$

$$\therefore R_T = R_1 + R_2 = \frac{3V_0^2 \sin 60^\circ \cos 60^\circ}{a} \quad (\text{still } 3R_1)$$

$$= \frac{3\sqrt{3}}{4} \frac{V_0^2}{a}$$ (where a is a gravitational acc. of the planet)

#17.



CM of the system will not move. Let x be the distance that CM of the boat moved (away from the shore). Then, (2.4 − X) is the distance that the dog moved closer to the shore.

$$m_b d_b = m_d d_d$$
$$18 \quad X = 4.5 \quad (2.4 - X)$$
$$X = 0.48\,m$$

So the doggie is $2.4 - 0.48 = 1.92\,m$ closer to the shore.

$$6.1 - 1.92 = \underline{4.18\,m}$$

#20.

$\theta_1 = 30°$

(a) cons. of p (in X direction)

$$\begin{array}{ccc} p_{ix} & & p_{fx} \\ m V_0 \sin\theta_1 & = & m V_f \sin\theta_2 \quad (V_0 = V_f) \end{array}$$

$$\therefore \theta_1 = \theta_2 = \underline{30°}$$

(b) $\Delta p_y = p_{fy} - p_{iy}$

$= -mV_0 \cos 30°$ (down) $- mV_0 \cos 30°$ (up)

$= -2mV_0 \cos 30°$

$= -0.5715767665\ kg\,m/sec$ ((−) means downward)

#27 Impulse $= \Delta p = \int F \cdot dt = (mV_f - mV_i)$

$\therefore F = \frac{dp}{dt}$ (rate of change of p)

Impulse given to a bullet (by Superman)

$\Delta p = m_b V_{bf} - m_b V_{bi} = 2 m_b V_{bf}$ (since $V_{bf} = -V_{bi}$)

Total impulse given to bullets per second

$\frac{\Delta p_T}{sec} = \#\ \text{bullets}/\text{sec} \cdot 2 m_b V_{bf}$

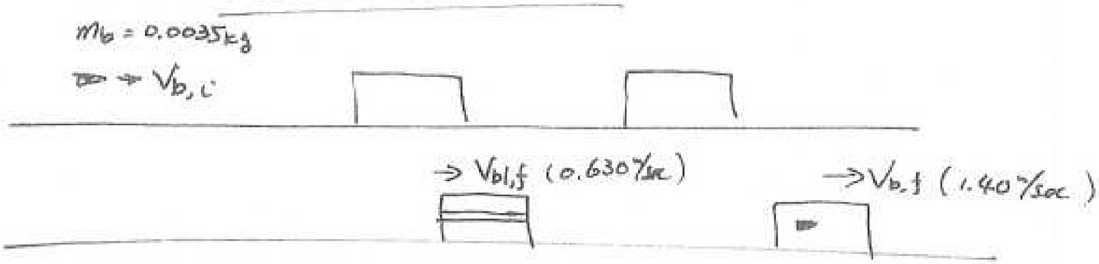
$= \frac{100\ \text{bullets}}{60\ \text{sec}} \cdot 2(0.003\text{kg})(-500\ \text{m/sec})$

$= -5N$ (Force given to opposite direction of original direction of bullets)

By Newton's third law, if we take Superman & bullets as a system, there is no net force. Superman must take equal & opposite reaction.

$\therefore$ $\underline{F_{\text{onto superman}} = 5N}$

#49.



(a) take p_i as the bullet between the blocks (We don't need to worry about p of m_1 since there is no change during the collision of the bullet and m_2)

$p_i \qquad\qquad p_f$

$m_b V_{b\ \text{between}} = (m_b + m_2) V_{b,f}$ (perfectly inelastic collision)

$V_{b,\text{between}} = \frac{m_b + m_2}{m_b} V_{b,f}$

$= \left(\frac{0.0035\text{kg} + 1.8\text{kg}}{0.0035\text{kg}}\right)(1.4\ \text{m/sec}) = \underline{721.4\ \text{m/sec}}$

(b) take p_i as the bullet before it hits m_1

p_f as the bullet embedded in block m_2

$\underline{p_i}$

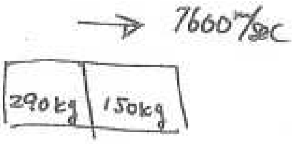
Don't forget this.

$$m_b V_{b,i} = m_1 V_{b1,f} + (m_b + m_2) V_{b2,f}$$

$$V_{bi} = \frac{m_1}{m_b} V_{b1,f} + \frac{m_b + m_2}{m_b} V_{b2,f}$$

$$= 216 \text{ m/sec} + 721.4 \text{ m/sec} = \underline{\underline{937.4 \text{ m/sec}}}$$

#85



m_R : mass of the rocket
m_C : " capsule
V_i : 7600 m/sec

Since there is no external force, p conserves.

$p_i \qquad\qquad p_f$

$$(m_R + m_C) V_i = m_R V_{Rf} + m_C V_{cf}$$

Also their relative speed is 910 m/sec

$$\rightarrow V_{cf} = V_{Rf} + 910$$

$$(m_R + m_C) V_i = m_R V_{Rf} + m_C (V_{Rf} + 910)$$

$$(m_R + m_C) V_i = m_R V_{Rf} + m_C V_{Rf} + 910 m_C$$

$$(m_R + m_C) V_i - 910 m_C = (m_R + m_C) V_{Rf}$$

$$V_{Rf} = \frac{(m_R + m_C) V_i - 910 m_C}{m_R + m_C}$$

$$= \underline{\underline{7289.772727 \text{ m/sec}}}$$

(a) & (b)

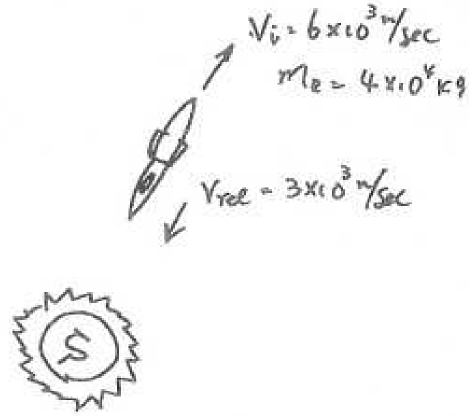
$$V_C = V_{Rf} + 910 = \underline{8199.772727 \text{ m/sec}}$$

(C) & (d) $KE_i = \frac{1}{2}(m_R + m_C) V_i^2 = \underline{\underline{1.27072 \times 10^{10}\ J}}$

$KE_f = \frac{1}{2} m_R V_{Rf}^2 + \frac{1}{2} m_C V_{Cf}^2 = \underline{\underline{1.27481349 \times 10^{10}\ J}}$

why $KE_i < KE_f$? the ΔE came from the spring energy transferred into KE.

#112



(a) $F = ma$

$= 4\times10^4\ kg \cdot 2\ m/sec^2$

$= \underline{\underline{8\times10^4\ N}}$

(b) this force (8×10^4 N) must come from the fuel.

Force per second :

Fuel / Rocket

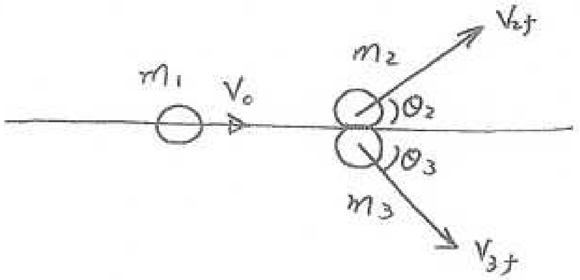
$$\frac{dp}{dt} = \frac{d(mv)}{dt} = \overbrace{V_{rel}\frac{dm}{dt}}^{\text{Fuel}} + \boxed{m\frac{dv}{dt}}$$

equal & opposite.

$$8\times10^4\ N = 3\times10^3\ m/sec\ \frac{dm}{dt}$$

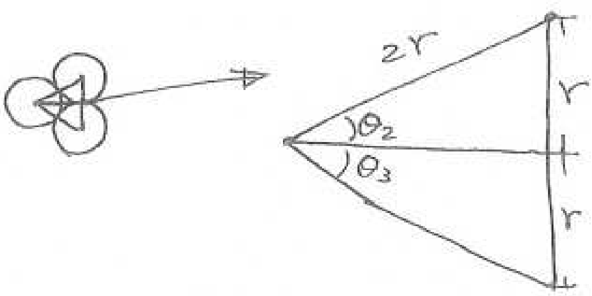
$$\frac{dm}{dt} = \frac{8\times10^4\ N}{3\times10^3\ m/sec} = \underline{\underline{2.\dot{6}\times10^1\ kg/sec}}$$

#130



$V_0 = 10\ m/sec$

$m_1 = m_2 = m_3 \equiv m$



$\sin^{-1}\frac{r}{2r} = \theta_2 = \theta_3$

$\Rightarrow \theta_2 = \theta_3 = 30° \equiv \theta$

Cons. of p

x

p_{ix}

$$m V_0 = m V_{1f} + m V_{2f} \cos\theta + m V_3 \cos\theta$$

$$V_0 = V_{1f} + V_{2f} \cos\theta + V_3 \cos\theta \quad \text{———} \; ①$$

y

$$p_{iy} = p_{fy}$$

$$0 = m V_{2f} \sin\theta - m V_{3f} \sin\theta$$

$$m V_{2f} \sin\theta = m V_{3f} \sin\theta$$

$$\therefore V_{2f} = V_{3f} = V_f \quad \text{———} \; ②$$

$① \leftarrow ②$

$$V_0 = V_{1f} + V_f \cos\theta + V_f \cos\theta$$

$$= V_{1f} + 2V_f \cos\theta$$

$$\therefore V_0 = V_{1f} + \sqrt{3}\, V_f \quad \text{———} \; ①'$$

Cons. of E

E_i

$$\frac{1}{2} m V_0^2 = \frac{1}{2} m V_{1f}^2 + \frac{1}{2} m V_{2f}^2 + \frac{1}{2} m V_{3f}^2$$

$$V_0^2 = V_{1f} + V_{2f}^2 + V_{3f}^2 \qquad (\text{But } V_{2f} = V_{3f} = V_f)$$

$$V_0^2 = V_{1f} + 2V_f^2 \quad \text{———} \; ③$$

Eqn. $①'$. solve for V_{1f}

$$V_{1f} = V_0 - \sqrt{3}\, V_f \quad ①'' \longrightarrow ③$$

$$V_0^2 = (V_0 - \sqrt{3}\, V_f)^2 + 2V_f^2$$

$$V_0^2 = V_0^2 - 2\sqrt{3}\, V_0 V_f + 3V_f^2 + 2V_f^2$$

$$0 = -2\sqrt{3}\, V_0 V_f + 5V_f^2$$

$$5V_f^2 = 2\sqrt{3}\, V_0 V_f$$

$$V_f = \frac{2\sqrt{3}}{5} V_0$$

(one possible sol. is $V_f = 0$
then $V_0 = V_{1f} = 10$ m/sec
→ as if m_1 went through ... but
that is not what we want)

$= \frac{2\sqrt{3}}{5} \cdot 10 = \underline{\underline{6.928203230 \text{ m/sec}}}$ —③'

①'' ← ③'

$V_{1f} = V_0 - \sqrt{3}\, V_f = \underline{-2 \text{ m/sec}}$

$V_{1f} = -2 \text{ m/sec}$ (moves backward)
$V_{2f} = V_{3f} = 6.928 \text{ m/sec}$ at $\pm 30°$

Additional problems: The topics put presented in this chapter used be presented in two chapters. There are more important problems to check your understanding. Make sure you do these problems as well as assigned homework questions.

#1 A railroad flatcar of weight W can roll without friction along a straight horizontal track. Initially, a man of weight w is standing on the car, which is moving to the right with sped v_0. What is the change in velocity of the car if the man runs to the left so that his speed relative to the car is v_{rel}?

#2 An 8.0 kg body is traveling at 2.0 m/sec with no external force acting on it. At a certain instant, an internal explosion occurs, splitting the body into two chunks of 4.0 kg mass each. The explosion gives that chunks an additional 16 J of kinetic energy. Neither chunk leaves the ling of original motion. Determine the speed and direction of motion of each of the chunks after the explosion.

#3 A 1500 kg automobile starts from rest on a horizontal road and gains a speed of 72 km/hr in 30 seconds. (a) What is the kinetic energy of the auto at the end of the 30 sec? (b) What is the average power required of the car during the 30 sec interval? (c) What is the instantaneous power at the end of the 30 sec interval, assuming the acceleration is constant?

#4 During a violent thunderstorm, hail of diameter 1.0 cm fall directly downward at a speed of 25 m/sec. There are estimated to be 120 hails per cubic meter of air. (a) What is the mass of each hailstone (density 0.92 g/cm^3)? (b) Assuming that the hail does not bounce, find the magnitude of the average force on a float room measuring 10 m X 20 m due onto the impact of the hail. (Hint: During impact, the force on a hailstone from the roof is approximately equal to the net force on the hailstone, because the gravitational force on it is small.)

#5 A ball having a mass of 150 g strikes a wall with a speed of 5.2 m/sec and rebounds with only 50% of its initial kinetic energy. (a) What is the speed of the ball immediately after rebounding? (b) What is the magnitude of the impulse on the wall from the ball? (c) If the ball was in contact with the wall for 7.6 ms, what was the magnitude of the average force on the ball from the wall during this time interval?

#6 Spacecraft Voyager 2 (of mass m and speed v relative to the Sun) approaches the planet Jupiter (of mass M and speed of V_J relative to the Sun). The spacecraft rounds the planet and departs in the opposite direction. What is its speed, relative to the Sun, after this slingshot encounter, which can be analyzed as a collision? Assume v = 12 km/sec and V_J = 13 km/sec (the orbital speed of Jupiter). The mass of Jupiter is much greater than the mass of the spacecraft (M>>m).

Solutions:

#1

W = weight of the car

w = weight of the man

v_0 = original velocity (ground velocity)

v_c = Final velocity of the car (ground velocity)

v_m = Final velocity of the man(ground velocity)

$$\vec{p}_i \qquad\qquad \vec{p}_f$$

$$\left(\frac{W+w}{g}\right)\vec{v}_0 = \left(\frac{W}{g}\right)\vec{v}_c + \left(\frac{w}{g}\right)\vec{v}_m \quad \text{but } \vec{v}_m = \vec{v}_c + \vec{v}_{rel}$$

$$\left(\frac{W+w}{g}\right)\vec{v}_0 = \left(\frac{W}{g}\right)\vec{v}_c + \left(\frac{w}{g}\right)(\vec{v}_c + \vec{v}_{rel})$$

$$(W+w)\vec{v}_0 = (W)\vec{v}_c + (w)(\vec{v}_c + \vec{v}_{rel})$$

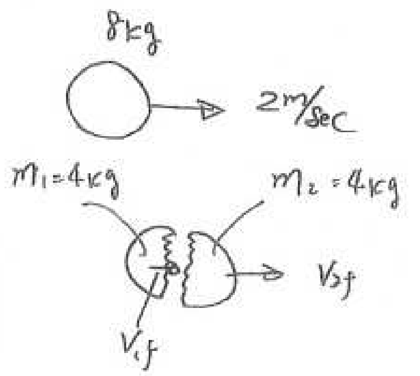
$$(W+w)\vec{v}_0 = (W+w)\vec{v}_c + (w)\vec{v}_{rel}, \text{ and } \Delta\vec{v} = \vec{v}_c - \vec{v}_0$$

$$(W+w)\vec{v}_c - (W+w)\vec{v}_0 = -w\vec{v}_{rel}$$

$$(W+w)(\vec{v}_c - \vec{v}_0) = -w\vec{v}_{rel}$$

$$\therefore \vec{v}_c - \vec{v}_0 = -\frac{w\vec{v}_{rel}}{W+w} = \Delta\vec{v}$$

#2



$$KE_i = \frac{1}{2} m V_i^2 = 16J$$

from explosion

$$KE_f = KE_i + 16J = 32J$$

Cons. of p

$$(m_1 + m_2) V_i = m_1 V_{1f} + m_2 V_{2f}$$

$$8 \cdot 2 = 4 V_{1f} + 4 V_{2f}$$

$$4 = V_{1f} + V_{2f}$$

$$\therefore V_{1f} = 4 - V_{2f} \quad \text{———} \; (1)$$

Cons. of E

$$KE_f = KE_i + 16J$$

$$\frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 = \frac{1}{2}(m_1 + m_2) V_i^2 + 16J$$

$$\frac{1}{2}\cdot 4\cdot V_{1f}^2 + \frac{1}{2}\cdot 4\cdot V_{2f}^2 = \frac{1}{2}(8)\,2^2 + 16\,J$$

$$2V_{1f}^2 + 2V_{2f}^2 = 32$$

$$V_{1f}^2 + V_{2f}^2 = 16 \quad \text{———} \quad ②$$

$$① \leftarrow ②$$

$$(4 - V_{2f})^2 + V_{2f}^2 = 16$$

$$16 - 8V_{2f} + V_{2f}^2 + V_{2f}^2 = 16$$

$$2V_{2f}^2 - 8V_{2f} = 0$$

$$2V_{2f}(V_{2f} - 4) = 0$$

$$V_{2f} = 0 \text{ or } 4\ \text{m/sec}$$

$$V_{1f} = 4\ \text{m/sec or } 0\ \text{m/sec}$$

$\Rightarrow$ One part becomes 0 m/sec & the other 4 m/sec

$$72\ \text{km/hr} = 72000\ \text{m}/3600\ \text{sec} = 20\ \text{m/sec}$$

#3

(a) $KE_f = \frac{1}{2} m V_f^2 = \frac{1}{2}(1500)(20)^2 = \underline{\underline{3.0\times10^5\ J}}$

(b) $\bar{P} = \frac{W}{t} = \frac{3.0\times10^5\ J}{30\ \text{sec}} = \underline{\underline{1.0\times10^4\ \text{watts}}}$

(c) $P = \frac{d(\int F\cdot dr)}{dt} = \frac{d(\int ma\,dr)}{dt}$

since a is const.,

$$= \frac{d(ma\int dr)}{dt}$$

$$= \frac{d(mar)}{dt}$$

$$= ma\frac{dr}{dt}$$

$$= ma\,V$$

$a = \text{const}$

$V = at + \cancel{V_0}^{\,0}$

$20\ \text{m/sec} = a\cdot 30\ \text{sec}$

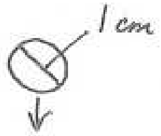
$a = \frac{2}{3}\ \text{m/sec}^2$

$$= 1500 \cdot \frac{2}{3} \cdot 20$$

$$= \underline{\underline{2 \times 10^4 \text{ watts}}}$$

make sure that you know the difference between $\overline{P}$ (average power) and P (instantaneous power).

#4



$D = 1\text{ cm} \rightarrow r = 0.5\text{ cm}$

$V_i = -25\text{ m/sec}$ (down)

(a) $m = \rho \cdot \text{vol}$

$= 0.92\text{ g/cm}^3 \cdot \frac{4}{3}\pi(0.5\text{ cm})^3 = \underline{\underline{0.48171087335\text{ g}}}$

(b) $\Delta p_{(\text{each hail})} = p_f - p_i$

$= 0 - m V_i$

$= -(0.4817\ \text{g})(\overset{\text{down}}{-25\text{ m/sec}})$

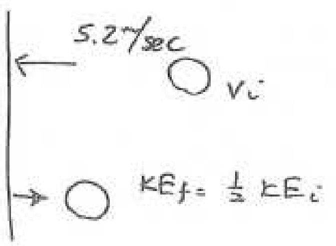
$= 1.204277184 \times 10^{-2}\text{ kg m/sec}$ ——— ①

How many hail stones per second do they hit the ground?

$120\text{ stones/m}^3 \cdot 10\text{ m} \times 20\text{ m} \times 25\text{ m} = 6 \times 10^5$ hails ——— ②

$\overline{F} = \Delta p/\text{hail} \times \frac{\#\text{ hails}}{\text{sec}} = ① \times ② \ \underline{\underline{= 7.225663104 \times 10^3\text{ N}}}$

#5



(a)

$KE_i = \frac{1}{2} m_b V_{ib}^2$

&

$KE_f = \frac{1}{2} KE_i = \frac{1}{2}\left(\frac{1}{2} m_b V_{ib}^2\right) = \frac{1}{2} m_b V_f^2$

$V_f = \sqrt{\frac{1}{2} V_{ib}^2} = \underline{\underline{3.676955262\text{ m/sec}}}$

(b)

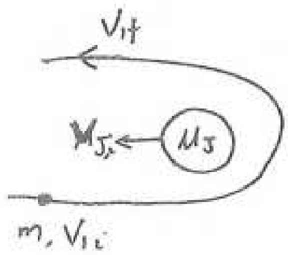
$$\Delta \vec{p} = m_b \vec{V_{bf}} - m_b \vec{V_{bi}}$$
$$= m_b (V_{bf} - V_{bi})$$
$$= m_b (3.676955\ldots - (\overset{\downarrow}{-}5.2))$$
$$= \underline{\underline{1.331543289 \text{ kg m/sec}}}$$

It was going "negative" direction.

(c)

$$F = \frac{\Delta p}{\Delta t} = \frac{1.3315 \text{ kg m/sec}}{7.6 \times 10^{-3} \text{ sec}} = \underline{\underline{175.2030644 \text{ N}}}$$

#6



$V_{1i} = 12$ km/sec

$V_{Ji} = 13$ km/sec

Collision w/o actual physical collision

→ perfectly elastic collision

(as we did in the lab w/ magnets)

⇒ Both p & E conserve.

$\underline{p}$ (I will use the direction of Jupiter as +)

$$\overbrace{M_J V_{Ji} - m V_{1i}}^{p_i} = \overbrace{M_J V_{Jf} + m V_{1f}}^{p_f}$$

$$V_{Jf} = \frac{M_J V_{Ji} - m V_{1i} - m V_{1f}}{M_J}$$

$$= V_{Ji} - \frac{m}{M_J}(V_{1i} + V_{1f})$$

For $m <<<< M_J$ $(m \sim 10^{2\sim4}\text{kg},\ M_J \sim 10^{26}\text{kg})$

$\sim V_{Ji}$ ⇒ $\boxed{\text{this indicates that speed of Jupiter will not change.}}$

Also.

$$M_J (V_{Ji} - V_{Jf}) = m (V_{1i} + V_{1f}) \quad \text{——— ①}$$

E

$$\frac{1}{2} M_J V_{Ji}^2 + \frac{1}{2} m V_{1i}^2 = \frac{1}{2} M_J V_{Jf}^2 + \frac{1}{2} m V_{1f}^2$$

$$M_J V_{Ji}^2 + m V_{1i}^2 = M_J V_{Jf}^2 + m V_{1f}^2$$

$$M_J \left(V_{Ji}^2 - V_{Jf}^2 \right) = m \left(V_{1f}^2 - V_{1i}^2 \right)$$

$$\underbrace{M_J \left(V_{Ji} - V_{Jf} \right)}_{①} \left(V_{Ji} + V_{Jf} \right) = m \left(V_{1f}^2 - V_{1i}^2 \right)$$

$$m \left(V_{1i} + V_{1f} \right) \left(V_{Ji} + V_{Jf} \right) = m \left(V_{1f}^2 - V_{1i}^2 \right) = m \left(V_{1f} + V_{1i} \right) \left(V_{1f} - V_{1i} \right)$$

$$V_{Ji} + V_{Jf} = V_{1f} - V_{1i}$$

$$\Rightarrow \quad V_{1f} = V_{Ji} + V_{Jf} + V_{1i} = 2 V_{Ji} + V_{1i}$$

$$= 2(13 \text{ km/sec})(12 \text{ km/sec}) = \underline{\underline{38 \text{ km/sec}}}$$

(217% increase w/o using fuel!)