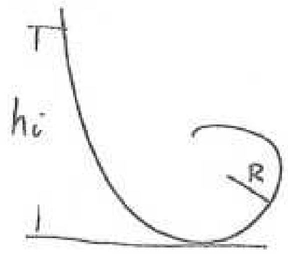
ch 11 # 8, 39, 47, 51, 56, 64, 100

8



Go back to Page 161 # 6 and identify similarities & differences

(a) On the top of the loop, its own weight is causing the centripetal force ($\Rightarrow N = 0$)

$$F_{centri} = mg = m\frac{V^2}{R}$$

$$V^2 = gR \quad \text{—— (1)}$$

Cons. of E

E_i $\qquad$ E_f

$$PE_i = PE_f + KE + RE$$

$$mgh_i = mgh_f + \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$$

$$\left[\begin{array}{l} I_{\text{solid sphere}} = \frac{2}{5}mr^2 \\ V = r\omega \Rightarrow \omega = \frac{V}{r} \end{array}\right]$$

$$mgh_i = mg(2R) + \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{V^2}{r^2}$$

This is one of the reasons why I've been stressing about upper & lower cases.

$$gh_i = 2gR + \frac{1}{2}V^2 + \frac{1}{5}V^2$$

$$gh_i = 2gR + \frac{7}{10}V^2 \quad \text{—— (2)}$$

Remember seeing this in the lab?

(2) ← (1)

$$gh_i = 2gR + \frac{7}{10}(gR)$$

$$\underline{\underline{h_i = \frac{27}{10}R}}$$

(b) E_i

$$PE_i = PE_f + KE + RE$$

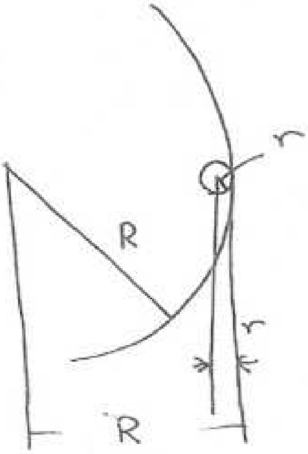
$$mg(6R) = mgR + \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$$

$$6mgR = mgR + \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\cdot\frac{V^2}{r^2}$$

$$5mgR = \frac{1}{2}mV^2 + \frac{1}{5}mV^2$$

$$5gR = \frac{7}{10}V^2$$

$$V^2 = \frac{50}{7}gR \quad \text{———} \quad (1)$$



$$F_{centri} = m\frac{V^2}{d}$$

d : dist. between the Center of the loop to the Center of the ball

$$= m\frac{V^2}{R-r} \quad \text{———} \quad (2)$$

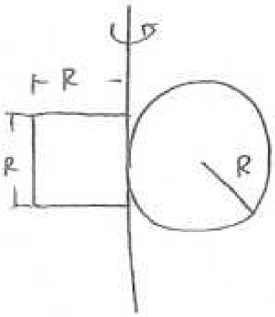
$(2) \leftarrow (1)$

$$= m\frac{\frac{50}{7}gR}{R-r}$$

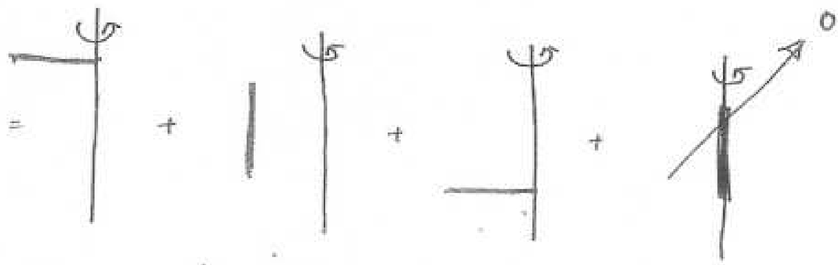
For $R >> r$

$$F = m\frac{\frac{50}{7}gR}{(R-r)} = \underline{\underline{\frac{50}{7}gm}}$$

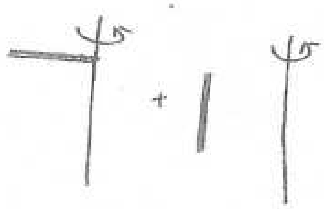
#39



I_{square} =

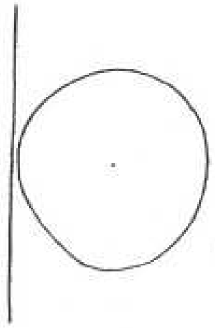


= 2 ×



$$= 2\times\frac{1}{3}mR^2 + mR^2$$

$$= \frac{2}{3}mR^2 + mR^2 = \underline{\frac{5}{3}mR^2}$$



I_{hoop}

$= I_{cm} + I_{\text{hoop WRT its own center}}$

$I_{\text{hoop W.R.T. its center}}$ is derived elsewhere

$(\phi) \quad = \frac{1}{2} m R^2$

$= mR^2 + \frac{1}{2} mR^2$

$= \underline{\underline{\frac{3}{2} m R^2}}$

$I_{Total} = I_{square} + I_{hoop}$

$= \frac{5}{3} mR^2 + \frac{3}{2} m R^2$

$= \frac{19}{6} m R^2 = \frac{19}{6} \, 2 \cdot (0.5)^2 = \underline{\underline{1.58\dot{3} \text{ kg m}^2}}$

#47

(a) they start rotating W.R.T. the center of rod, 1.5 m away from them.
At it was shown in # 27, $\vec{\ell} = m v d$

$\ell_i = \ell_f$

$m v d = I \omega \qquad \left(I = 2 \times m r^2 = 2 \times m \left(\frac{1}{2} d\right)^2 = \frac{1}{2} m d^2 \right)$

$\omega = \frac{m v d}{I}$

$= \frac{m v d}{\frac{1}{2} m d^2} = \frac{1.4}{\frac{1}{2} \cdot 3} = \underline{\underline{0.9\dot{3} \text{ rad/sec}}}$

(b)

$KE = 2 \cdot \frac{1}{2} m v^2 = 50 . (1.4)^2 = \underline{\underline{98 J}}$

$RE = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} m d^2\right) . (0.9\dot{3})^2 = \underline{\underline{98 J}}$] → Energy Conserves!

(c) Cons. of ℓ

$$I_i \omega_i = I_f \omega_f$$

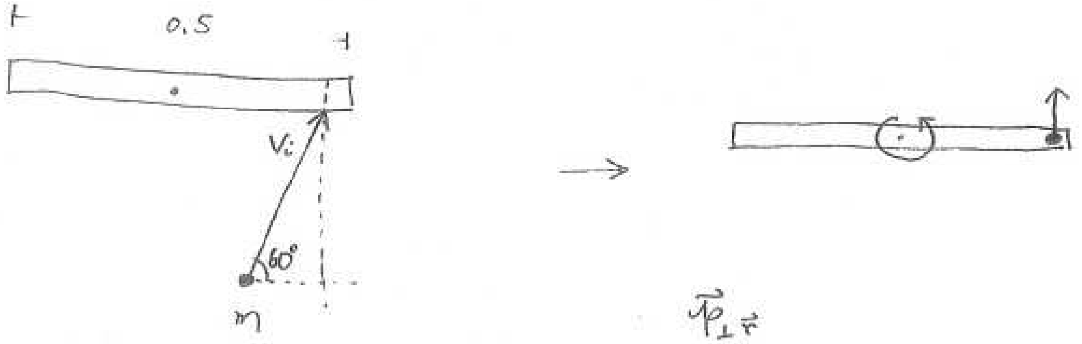
$$\omega_f = \frac{I_i \omega_i}{I_f} = \underline{\underline{8.4 \text{ rad/sec}}}$$

$$\left[\begin{array}{ll} I_i = \frac{1}{2} m d_i^2 & d_i = 3m \\ I_f = \frac{1}{2} m d_f^2 & d_f = 1m \\ \omega_i = 0.9\dot{3} \text{ rad/sec} & \end{array}\right]$$

(d) $RE = \frac{1}{2} I_f \omega_f^2 = \underline{881.9 \text{ J}}$

(e) the diff. comes from skaters. they did work by pulling themselves toward center.

#51



$$\vec{\ell}_i = \vec{r} \times \vec{p} = 0.25 \cdot \overbrace{m_b V_i \sin 60°}^{\vec{p}_{\perp \vec{r}}}$$

$$\vec{\ell}_f = I_{rod}\ \omega_f + I_{bullet}\ \omega_f$$

$$= \frac{1}{12} M_R L_R^2 \cdot \omega_f + m_b \left(\frac{1}{2} L_R\right)^2 \cdot \omega_f$$

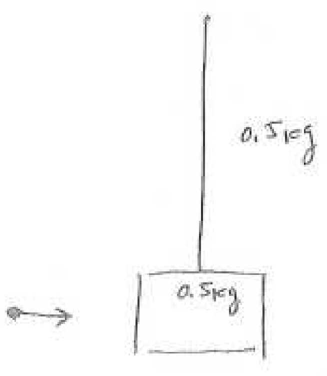
$$\vec{\ell}_i = \vec{\ell}_f$$

$$0.25\, m_b V_i \sin 60° = \frac{1}{12}(4)(0.5)^2 \cdot 10 + 0.003\left(\frac{1}{2}\, 0.5\right)^2 \cdot 10$$

$$V_i = \frac{\frac{1}{12}(4)(0.5)^2 \cdot 10 + 0.003\left(\frac{1}{2}\, 0.5\right)^2 \cdot 10}{0.25 \cdot 0.003 \sin 60°}$$

$$= \underline{\underline{1.29 \times 10^3 \text{ m/sec}}} \quad (1.294547604 \times 10^3 \text{ m/sec})$$

#56



$I_{rod} = 0.06 \text{ kg m}^2$

(a) $I_T = I_{Rod} + I_{block} + I_{bullet}$

$= 0.06\ kgm^2 + m_{block} r^2 + m_{bullet} \cdot r^2$

$= [0.06 + 0.5(0.6)^2 + (0.001)(0.6)^2]\ kg\,m^2$

$= \underline{\underline{0.24036\ kg\,m^2}}$

(b) Cons. of $\vec{l}$

$\vec{l}_i = \vec{l}_f$

$\vec{r} \times m_{bullet} V = I\,\omega_f$

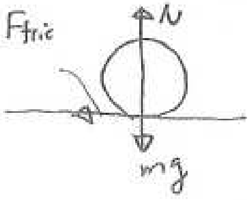
$V = \frac{I_T \omega_f}{r \cdot m_{bullet}} = \frac{0.24036 \cdot 4.5}{0.6 \cdot 0.001} = \underline{\underline{1.8027 \times 10^3\ m/sec}}$

#64

(a) $V_{com,f} = R\,\omega_f$

$= \underline{\underline{(0.11)\,\omega_f}}$ (clockwise)

(b) linear acc. is due to kinetic friction.



x	y
$-F_{fric} = -ma$	$N - mg = 0$
$-\mu_k N = -ma$	$N = mg$

$a = \frac{\mu_k mg}{m} = \mu_k g = \underline{\underline{2.0601\ m/sec^2}}$ (to negative x direction)

(c) the kinetic friction is causing the ball to rotate

⇒ kinetic friction (Force) causes "Torque".

$\tau = I\alpha = R F_{fric}$

$\alpha = \frac{R F_{fric}}{I} = \frac{\cancel{R} \mu_k \cancel{m} g}{\frac{2}{5} \cancel{m} R^{\cancel{2}}} = \frac{(0.21)(9.81)}{\frac{2}{5}(0.11)} = \underline{\underline{46.8204545\ rad/sec^2}}$

(clockwise)

(d) there are two ways to solve

(I) While the ball is slowing down linearly (question (b)), it gains its angular speed (question (c)). Since we know how to convert from linear to angular (or vice versa), we set their speeds to be the same.

Linear

$a = -2.0601 \ m/sec^2$ ——— (1)

$V = \int a \cdot dt = -2.0601t + \cancel{V_0}\ 8.5$ ——— (2)

$x = \int V \cdot dt = -2.0601 \cdot \frac{1}{2}t^2 + 8.5t + \cancel{x_0}\ 0$ ——— (3)

Angular (see (c))

$\alpha = \frac{R F_{fric}}{I} = \frac{\mu_k g}{\frac{2}{5}R}$ ——— (4)

$\omega = \int \alpha \cdot dt = \frac{\mu_k g}{\frac{2}{5}R} t + \cancel{\omega_0}\ 0$ ——— (5)

$\theta = \int \frac{\mu_k g}{\frac{2}{5}R} t \cdot dt = \frac{1}{2}t^2 \frac{\mu_k g}{\frac{2}{5}R}$ ——— (6)

Eqn (2) change it to ω

$V = \omega R \Rightarrow \omega = \frac{V}{R} = \frac{-2.0601t + 8.5}{R}$ ——— (2)'

their final speed should be the same

$\therefore$ (2)' = (5)

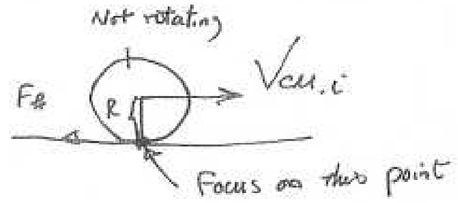
$$\frac{-2.0601t + 8.5}{R} = \frac{\mu_k g}{\frac{2}{5}R} t$$

$$\frac{8.5}{R} = \frac{\mu_k g t}{\frac{2}{5}R} + \frac{2.0601}{R} t = \left(\frac{\mu_k g}{\frac{2}{5}R} + \frac{2.0601}{R}\right) t$$

$$\therefore t = \frac{8.5}{R\left(\frac{\mu_k g}{\frac{2}{5}R} + \frac{2.0601}{R}\right)} = \frac{8.5}{0.11\left(\frac{0.21 \cdot 9.81}{\frac{2}{5} \cdot 0.11} + \frac{2.0601}{0.11}\right)}$$

$$= \underline{\underline{1.17880943 \text{ sec}}} \text{ ——— (2)''}$$

(II) The second way (and the way I like better personally) is to use the "Conservation of $\vec{\ell}$":



ℓ_i ← Due to its linear motion

ℓ_f ← due to its linear motion ← due to its angular motion

$$I\,\omega_{(\text{of the ball})} = m V_{cm,f}\, R + I\,\omega_f$$

$$m V_{cm,i} \cdot R = m V_{cm,f}\, R + \frac{2}{5} m R^2 \cdot \omega_f \leftarrow \omega_f = \frac{V_{cm,f}}{R}$$

$$V_{cm,i} = V_{cm,f} + \frac{2}{5} V_{cm,f}$$

$$V_{cm,i} = \frac{7}{5} V_{cm,f} \Rightarrow V_{cm,f} = \frac{5}{7} V_{cm,i}$$

$$\boxed{\omega_f = \frac{V_{cm,f}}{R} = \frac{\frac{5}{7} V_{cm,i}}{R}}$$

Also, $\omega_f = \alpha t = \dfrac{\mu_k g}{\frac{2}{5} R}\, t$ (see (c))

$$\therefore \omega_f = \frac{\frac{5}{7} V_{cm,i}}{R} = \frac{\mu_k g}{\frac{2}{5} R}\, t$$

$$t = \frac{\frac{5}{7} V_{cm,i} \cdot \frac{2}{5} R}{R \cdot \mu_k g} = \frac{\frac{2}{7} V_{cm,i}}{\mu_k g}$$

$$= \frac{\frac{2}{7} \cdot 8.5}{0.21 \cdot 9.81} = \underline{\underline{1.178860943 \text{ sec}}}$$

(e) ③ ← ②"

$$X = -2.0601 \cdot \tfrac{1}{2} t^2 + 8.5 t$$
$$= \underline{\underline{8.588531253 \text{ m}}}$$

(f) ② ← ③"

$$V = -2.0601\, t + 8.5 = \underline{\underline{6.071534693 \text{ m/sec}}}$$

#100

$$\vec{\tau} = \vec{R} \times \vec{F} = I \cdot \alpha$$

$$\therefore \int \tau \cdot dt$$

$$\int \vec{r} \times \vec{F} \cdot dt = rFt \quad \text{(both } r \text{ \& } F \text{ are const.)}$$

$$\int I \cdot \alpha \cdot dt = I \int \alpha \cdot dt = I\ \omega \Big|_{\omega_i\ (t=i)}^{\omega_f\ (t=f)} = I(\omega_f - \omega_i)$$

Additional Problems Ch. 11.

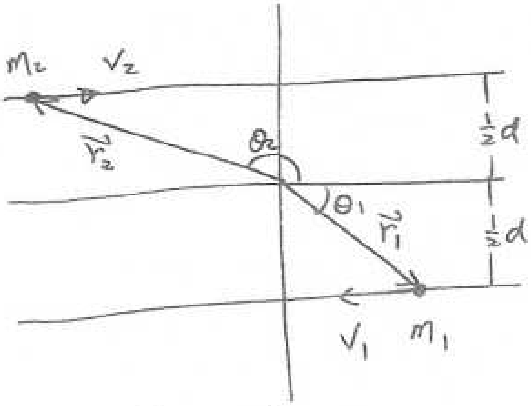
#1 Two particles, each of mass m and speed v travel in opposite directions along parallel lines separated by a distance d. (a) In terms of m, v, and d, find an expression for the magnitude L of the angular momentum of the two-particle system around a point midway between the two lines. (b) Does the expression change if the point about which L is calculated is not midway between the lines? (c) Now reverse the direction of travel for one of the particles and repeat (a) and (b).

#2 Two cylinders having radii R_1 and R_2 and rotational inertias I_1 and I_2 about their central axes are supported by axles perpendicular to the plane. The large cylinder is initially rotating clockwise with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually, slipping ceases, and the two cylinders rotate at constant rates in opposite directions. Find the final angular velocity ω_2 of the small cylinder in terms of I_1, I_2, R_1, R_2, and ω_0.

#3 A wheel is rotating freely at angular speed 800 rev/min of a shaft whose rotational inertial is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first, is suddenly coupled to tht same shaft. (a) What is the angular speed of the resultant combination of the shaft and two wheels? (b)What fraction of the original rotational kinetic energy is lost?

#1

(a)



Note:
I am using abs. values for α's and taking care of signs by realizing the directions of angular momenta

$$\vec{l}_T = \vec{l}_1 + \vec{l}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 = m_1(\vec{r}_1 \times \vec{v}_1) + m_2(\vec{r}_2 \times \vec{v}_2)$$

[I like clockwise motion positive. The book shows clockwise negative]

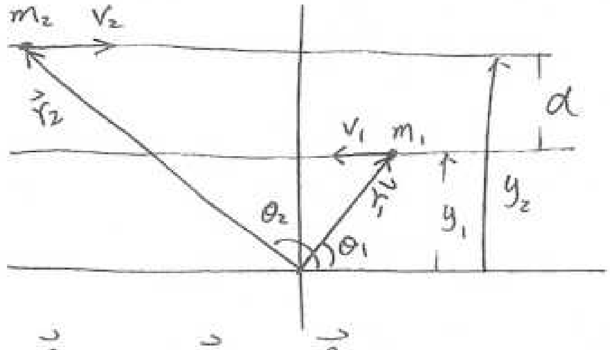
$$= m_1(r_1 V_1 \sin\theta_1) + m_2(r_2 V_2 \sin\theta_2) \qquad r_1 \sin\theta_1 = \tfrac{1}{2}d$$

$$= m V(\tfrac{1}{2}d) + m V(\tfrac{1}{2}d) \qquad r_2 \sin\theta_2 = \tfrac{1}{2}d$$

$$m_1 = m_2$$

$$= \underline{\underline{m v d}} \qquad |V_1| = |V_2|$$

(b)



$$\vec{\ell}_T = \vec{\ell}_1 + \vec{\ell}_2$$

$$= \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$= -m_1(\vec{r}_1 \times \vec{v}_1) + m_2(\vec{r}_2 \times \vec{v}_2) \qquad m_1 = m_2$$

counterclockwise   clockwise

$$= -m(r_1 V_1 \sin\theta_1) + m(r_2 V_2 \sin\theta_2) \qquad r_1 \sin\theta_1 = y_1$$

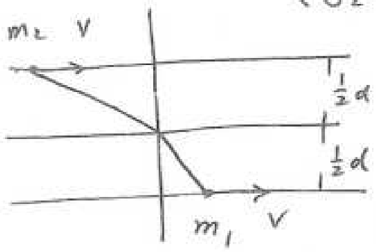
$$r_2 \sin\theta_2 = y_2$$

$$= -m v y_1 + m v y_2 \qquad |V_1| = |V_2|$$

$$= m v (y_2 - y_1) = \underline{\underline{m v d}}$$

<u>The same result as (a)</u>

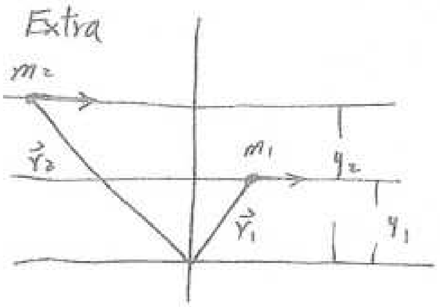
(c)



The set up eqn. is same as (a), except for a sign of $\vec{\ell}_1$ because it is rotating counterclockwise W.R.T. the origin

$$\vec{\ell}_T = -m v \tfrac{1}{2}d + m v \tfrac{1}{2}d = \underline{\underline{0}}$$

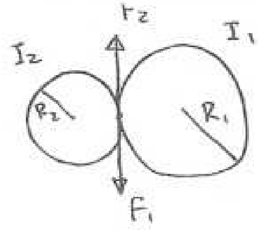
(d) Extra



$$\vec{\ell} = m v y_1 + m v y_2$$

$$= \underline{\underline{m v (y_1 + y_2)}}$$

#2



F_1 is a frictional force working on a larger wheel (so that it will slow down)

F_2 is a frictional force working on a smaller wheel (so that it will speed up)

F_1 & F_2 are equal and opposite and they exist until the wheels' linear vels. at contact pt. are equal (& opposite)
(angular speeds are <u>not</u> equal & opposite)

$$\tau_1 = \vec{R}_1 \times \vec{F}_1 \qquad \tau_2 = \vec{R}_2 \times \vec{F}_2$$

$$\Delta \ell = \int \tau_1 \, dt \qquad \Delta \ell = \int \tau_2 \, dt$$

counterclockwise

$$= R_1 \cdot (-F)\, t \qquad = R_2 F t$$

$$I_1(\omega_{1f} - \omega_{1i}) = -R_1 F t \quad — ① \qquad I_2(\omega_{2f} - \cancel{\omega_{2i}}^{\,0}) = R_2 F t$$

$$t = \frac{I_2 \omega_{2f}}{R_2 F} \quad — ③$$

① ← ②

$$I_1(\omega_{1f} - \omega_{1i}) = -R_1 F \frac{I_2 \omega_{2f}}{R_2 F}$$

$$I_1(\omega_{1f} - \omega_{1i}) = -\frac{R_1}{R_2} I_2 \omega_{2f}$$

Also, at the contact pt., their linear speeds should be the same: $V = R_1 \omega_{1f} = R_2 \omega_{2f}$

$$\therefore \omega_{1f} = \frac{R_2}{R_1} \omega_{2f}$$

$$I_1\left(\frac{R_2}{R_1}\omega_{2f} - \omega_{1i}\right) = -\frac{R_1}{R_2} I_2 \omega_{2f}$$

$$\frac{R_2}{R_1} I_1 \omega_{2f} - I_1 \omega_{1i} = -\frac{R_1}{R_2} I_2 \omega_{2f}$$

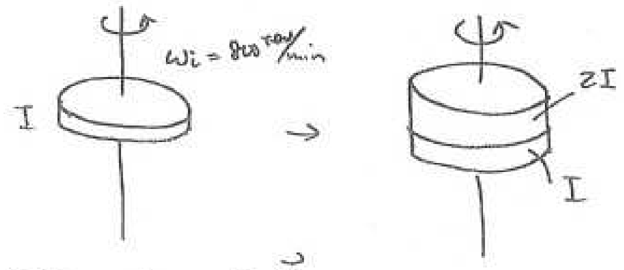
$$\frac{R_2}{R_1} I_1 \omega_{2f} + \frac{R_1}{R_2} I_2 \omega_{2f} = I_1 \omega_{1i}$$

$$\omega_{2f}\left(\frac{R_2}{R_1} I_1 + \frac{R_1}{R_2} I_2\right) = I_1 \omega_i$$

$$\omega_{2f}\left(\frac{R_2^2 I_1 + R_1^2 I_2}{R_1 R_2}\right) = I_1 \omega_i$$

$$\omega_{2f} = \left(\frac{R_1 R_2}{R_2^2 I_1 + R_1^2 I_2}\right) I_1 \omega_i$$

#3



(a) Cons of $\vec{\ell}$

$$\vec{\ell}_i = \vec{\ell}_f$$

$$I_1 \omega_i = I_1 \omega_f + I_2 \omega_f \qquad (I_2 = 2I_1)$$

$$\therefore\ I_1 \omega_i = (I_1 + 2I_1)\omega_f$$

$$\omega_f = \frac{I_1 \omega_i}{3I_1} = \frac{1}{3}\ 800\ rev/min = \underline{\underline{2.\dot{6} \times 10^2\ rev/min}}$$

$$= \underline{\underline{27.925268\ rad/sec}}$$

(b)

$$RE_i = \frac{1}{2} I_1 \omega_i^2$$

$$RE_f = \frac{1}{2} I_1 \omega_f^2 + \frac{1}{2} I_2 \omega_f^2$$

$$= \frac{1}{2} I_1 \omega_f^2 + \frac{1}{2} (2I_1)\omega_f^2$$

$$= \frac{1}{2}(3I_1)\ \omega_f^2 \qquad \left(\omega_f = \frac{1}{3}\omega_i^2 \text{ from (a)}\right)$$

$$= \frac{1}{2}(3I_1)\left(\frac{1}{3}\omega_i\right)^2$$

$$= \frac{1}{6} I_i \omega_i^2$$

$$\frac{\Delta RE}{RE_i} = \frac{\frac{1}{2} I_1 \omega_i^2 - \frac{1}{6} I_i \omega_i^2}{\frac{1}{2} I \omega_i^2} = \underline{\underline{\frac{2}{3}}} \quad \text{(It lost } \tfrac{2}{3} \text{ of the original E)}$$