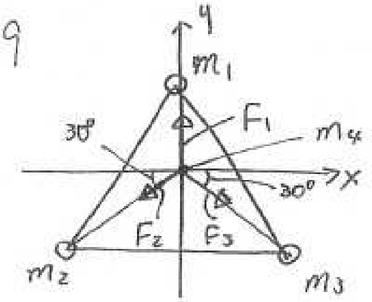
Ch 13 # 9, 32, 42, 103

#9



This setting is exactly the same as the Lab #1, first set up (3 forces separated by 120° from each)
So, $\Sigma\vec{F} = 0$, and $|\vec{F}_1| = |\vec{F}_2| = |\vec{F}_3|$ is what we expect to see.

$$F_1 = G\frac{m_1 m_4}{r^2}(0\hat{i} + 1\hat{j})$$

$$F_2 = G\frac{m_2 m_4}{r^2}(-\cos 30°\hat{i} - \sin 30°\hat{j})$$

$$F_3 = G\frac{m_3 m_4}{r^2}(\cos 30°\hat{i} - \sin 30°\hat{j})$$

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

(a) <u>x comp</u>

$$F_{1x} + F_{2x} + F_{3x} = 0 + G\frac{m_2 m_4}{r^2}(-\cos 30°) + G\frac{m_3 m_4}{r^2}(\cos 30°) = \underline{\underline{0}}$$

<u>y comp</u>

$$F_{1y} + F_{2y} + F_{3y} = G\frac{m_1 m_4}{r^2} + G\frac{m_2 m_4}{r^2}(-\sin 30°) + G\frac{m_3 m_4}{r^2}(-\sin 30°) = 0$$

$$m_2 = m_3 \equiv m$$

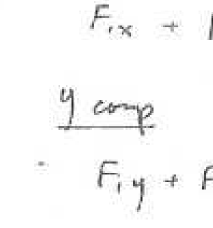
$$= G\frac{m_1 m_4}{r^2} - 2\frac{m m_4}{r^2}\sin 30°$$

$$\therefore G\frac{m_1 m_4}{r^2} = 2\frac{m m_4}{r^2}\cancel{\sin 30°}\ \tfrac{1}{2}$$

$$\underline{\underline{m_1 = m}}$$

(b) magnitude of each force will be double, but F_{net} is still "zero".

#32



$M = 5 \times 10^{23}$ kg
$R = 3 \times 10^6$ m

E_i $\qquad\qquad$ E_f

$$KE_i = KE_f + PE$$

$$\therefore KE_f = KE_i - PE$$

(a)

$$KE_f = KE_i - \int_{3\times10^6}^{4\times10^6} G\frac{m_1 m_2}{r^2}\cdot dr$$

(Don't ever think that PE is always equal to mgh!)

$$= 5\times10^7 J - 2.78025\times10^7 J$$

$$= \underline{\underline{2.21975\times10^7 J}}$$

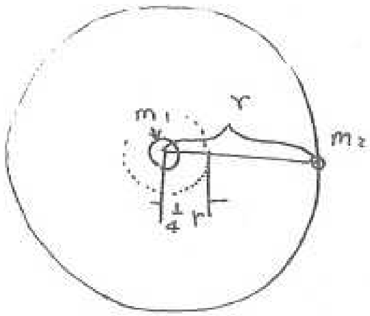
(b)

$$\overset{E_i}{KE} = \overset{E_f}{PE}$$

$$PE = \int_{3\times10^6}^{8\times10^6} G\frac{m_1 m_2}{r^2}dr = Gm_1m_2\left(\frac{1}{3\times10^6} - \frac{1}{8\times10^6}\right)$$

$$= \underline{\underline{6.950625\times10^7 J}}$$

#42



$$F = G\frac{m_1 m_2}{r^2} = m_2\frac{V^2}{r}$$

$$\therefore V = \sqrt{\frac{Gm_1}{r}}$$

$$P = \frac{2\pi r}{V} = \frac{2\pi r}{\sqrt{\frac{Gm_1}{r}}} = \sqrt{\frac{4\pi^2 r^3}{Gm_1}} = \sqrt{\frac{4\pi^2}{Gm_1}}\cdot r^{3/2}$$

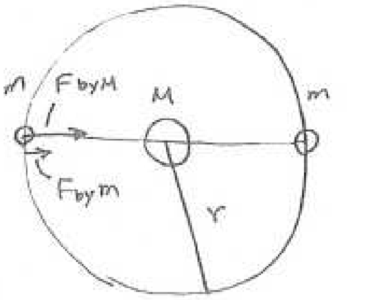
$\Rightarrow$ Orbital Period $\propto r^{3/2}$

Now radius $= \frac{1}{2}r$

$$P \propto \left(\frac{1}{2}r\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2} r^{3/2}$$

$$= \frac{1}{\sqrt{8}} r^{3/2} \Rightarrow \underline{\underline{\frac{1}{\sqrt{8}} \text{ Lunar month}}}$$

#103



$$F_{net} = F_{by\,M} + F_{by\,m}$$

$$= G\frac{mM}{r^2} + G\frac{mm}{(2r)^2}$$

$$= G\frac{mM}{r^2} + G\frac{m^2}{4r^2}$$

$$= \frac{Gm}{4r^2}(4M+m) = F_{centri} = m\frac{V^2}{r}$$

$$\therefore \frac{G\cancel{m}}{4r^{\cancel{2}}}(4M+m) = \cancel{m}\frac{V^2}{\cancel{r}}$$

$$\therefore V = \sqrt{\frac{G}{4r}(4M+m)}$$

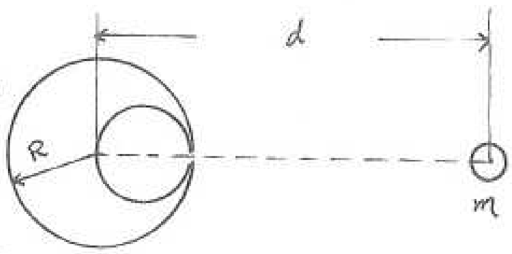
$$P = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{G}{4r}(4M+m)}} = \sqrt{\frac{4\pi^2 r^2}{\frac{G}{4r}(4M+m)}}$$

$$= 4\pi\sqrt{\frac{r^3}{G(4M+m)}}$$

$$= \underline{\underline{4\pi\left(\frac{r^3}{G(4M+m)}\right)^{1/2}}}$$

Ch. 13 additional problems:

#1 The diagram on the right shows a spherical hollow inside a lead sphere of radius R; the surface of the hollow passes through the center of the sphere and "touches" the right side of the sphere. The mass of the sphere before hollowing was M. With what gravitational force does the hollowed-out lead sphere attract a small sphere of mass m that lies at a distance d from the center of the lead sphere, on the straight line connecting the centers of the sphere and of the hollow?



#2 One model for a certain planet has a core of radius R and mass M surrounded by an outer shell of inner radius R, outer radius 2R, and mass 4M. If $M = 4.1 \times 10^{24}$ kg and $R = 6.0 \times 10^{6}$ m, what is the gravitational acceleration of a particle at points (a) R and (b) 3R from the center of the planet?

#3 A uniform solid sphere of radius R produces a gravitational acceleration of a_g on its surface. At what two distances from the center of the sphere is the gravitational acceleration $a_g/3$?

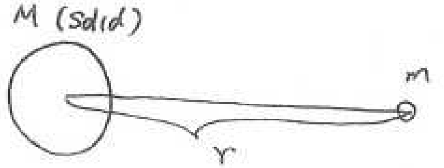
#4 A projectile is fired vertically from Earth's surface with an initial speed of 10 km/sec. Neglecting air drag, how far above the surface of Earth will it go?

#5 Three identical stars of mass M are located at the vertices of an equilateral triangle with side L. At what speed must they move if they all revolve under the influence of one another's gravitational force in a circular orbit circumscribing the triangle while still preserving the equilateral triangle?

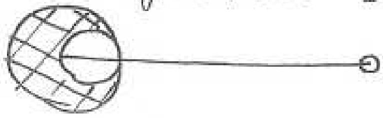
#6 Show that if an object is in an elliptical orbit with semi-major axis about a planet of mass M, then its distance r from the planet and speed v are related by $v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$

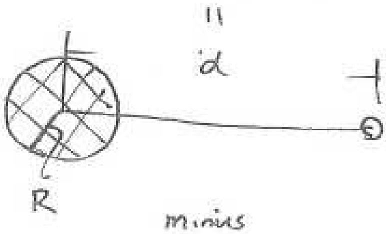
#7 One way to attack a satellite in Earth orbit is to launch a swarm of pellets in the same orbit as the satellite but in the opposite direction. Suppose a satellite in a circular orbit 500 km above Earth's surface collides with a pellet having mass 4.0 g. (a) What is the kinetic energy of the pellet in the reference frame of the satellite just before the collision? (b) What is the ratio of this kinetic energy to the kinetic energy of a 4.0 g bullet from a modern army rifle with a muzzle speed of 950 m/sec?

#1



How can we figure out the gravitational force when the object is not a nice shape? One way is to find the cm of the object, but I will use another way.

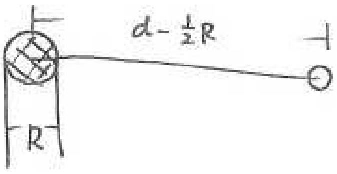




$\Rightarrow \quad G\dfrac{Mm}{d^2}$

minus

minus



$\Rightarrow \quad G\dfrac{M'm}{(d-\frac{1}{2}R)^2}$ $\quad M' \equiv$ mass of the pushed out (small) sphere.

M'?

$M' = \rho \cdot vol$ where $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$ & $vol = \frac{4}{3}\pi(\frac{1}{2}R)^3$

$\therefore M' = \dfrac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi\left(\frac{1}{2}R\right)^3$

$= \underline{\frac{1}{8}M} \quad \Rightarrow \quad G\dfrac{\frac{1}{8}Mm}{(d-\frac{1}{2}R)^2}$ (Force by the small sphere)

$$F_{net} = G\frac{Mm}{d^2} - G\frac{Mm}{8(d-\frac{1}{2}R)^2}$$

$$= GMm\left(\frac{1}{d^2} - \frac{1}{8(d-\frac{1}{2}R)^2}\right)$$

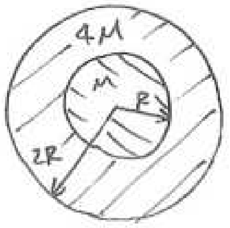
$$= GMm\left(\frac{8(d-\frac{1}{2}R)^2 - d^2}{8d^2(d-\frac{1}{2}R)^2}\right)$$

$$= GMm\left(\frac{8d^2 - 8dR + 2R^2 - d^2}{8d^2(d-\frac{1}{2}R)^2}\right)$$

$$= GMm\left(\frac{7d^2 - 8dR + 2R^2}{8d^2(d-\frac{1}{2}R)^2}\right)$$

I don't know which one is a simpler form for an answer?

#2



$M = 4.1 \times 10^{24}\ kg$

$R = 6 \times 10^{6}\ m$

(a) $r = R$

$$a = \frac{G\, M_{\text{inside of } r}}{r^2}$$

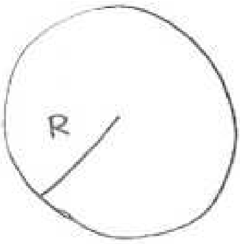
$$= \frac{G\ 4.1 \times 10^{24}}{(6 \times 10^6)^2} = \underline{\underline{7.59935\ m/sec^2}}$$

(b) $r = 3R$

$$a = \frac{G\, M_{\text{inside of } r}}{r^2} = \frac{G\ (4M + M)}{(3R)^2} = \frac{5\, GM}{9\, R^2} = \frac{5}{9} \text{ of } (a)$$

$$= \underline{\underline{4.22186111\ m/sec^2}}$$

#3



Since a is the largest at surface, a decreases both inward & outward.

$$a_{inside} = G \frac{M(\text{inside of } r)}{r^2} = G \frac{\rho \cdot Vol(\text{inside of } r)}{r^2}$$

$$= G \frac{\frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3}{r^2}$$

$$= \frac{GM}{R^3} \cdot r$$

$$a = \frac{1}{3} \frac{GM}{R^2} = \frac{GM}{R^3} \cdot r$$

$$\underline{\underline{r = \frac{1}{3} R}}$$

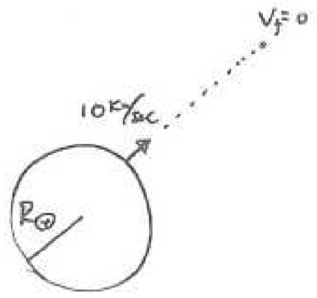
outside

$$a = \frac{1}{3} \frac{GM}{R^2} = \frac{GM}{r^2}$$

$$r^2 = 3R^2$$

$$\underline{\underline{r = \sqrt{3}\, R}}$$

#4



$R_\oplus = 6.37 \times 10^6 \, m$

$M_\oplus = 5.98 \times 10^{24} \, kg$

$V_0 = 1 \times 10^4 \, m/sec$

$V_f = 0 \, m/sec$

$E_i \qquad E_f$

$KE = PE$

$$\frac{1}{2} m V_i^2 = \int_{R_\oplus}^{r} G \frac{m M_\oplus}{r^2} \cdot dr$$

$$\frac{1}{2} m V_i^2 = G m M_\oplus \left(\frac{1}{R_\oplus} - \frac{1}{r} \right)$$

$$\frac{V_i^2}{2 G M_\oplus} = \frac{1}{R_\oplus} - \frac{1}{r}$$

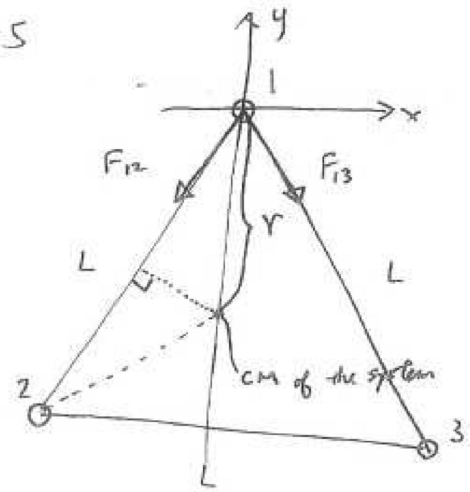
$$\frac{1}{r} = \frac{1}{R_\oplus} - \frac{V_i^2}{2 G M_\oplus}$$

$$r = \frac{1}{\frac{1}{R_\oplus} - \frac{V_i^2}{2 G M_\oplus}} = 3.156632029 \times 10^7 m$$

So, Above the ground is

$r - R_\oplus = \underline{\underline{2.519632029 \times 10^7 m}}$

#5



At star 1

$\Sigma F_x = 0 \qquad |F_{12}| = |F_{13}| = G \frac{MM}{L^2}$

$\Sigma F_y = -F_{12} \cos 30° - F_{13} \cos 30°$

$= -2 \, G \frac{M^2}{L^2} \cos 30°$

Also,

$$\left\{ \begin{array}{l} r \cos 30° = \frac{1}{2} L \\ r = \frac{L}{2 \cos 30°} \end{array} \right\}$$

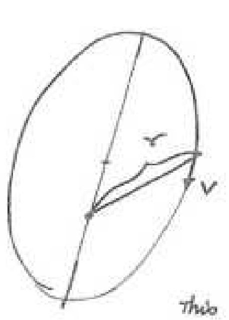
$\Sigma F_y = F_{centri}$

$$-2G \frac{M^2}{L^2} \cos 30° = -M \frac{V^2}{r}$$

$$2G \frac{M}{L^2} \cos 30° = \frac{V^2}{\frac{L}{2 \cos 30°}} = \frac{V^2 \, 2 \cos 30°}{L}$$

$$\frac{GM}{L} = V^2 \Rightarrow \therefore \underline{\underline{V = \sqrt{\frac{GM}{L}}}}$$

6



$$E_{Total} = KE + PE$$

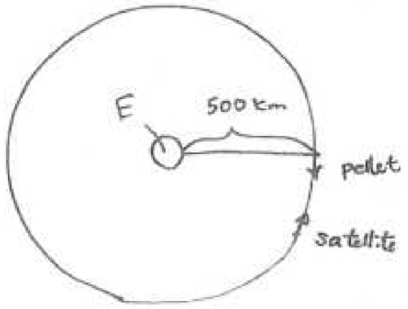
$$-\frac{GMm}{2a} = \frac{1}{2}mV^2 - G\frac{Mm}{r}$$

This is proven on P 346.

$$\frac{1}{2}mV^2 = GmM\left(\frac{1}{r} - \frac{1}{2a}\right)$$

$$\therefore V^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

7



$$F_{grav} = G\frac{M_{\oplus} m}{r^2} = m\frac{v^2}{r}$$

$$V = \sqrt{\frac{GM_{\oplus}}{r}}$$

Both satellite & pellet have the same speed But opposite direction → WRT the satellite, the pellet has twice the speed calculated above.

(a)
$$KE = \frac{1}{2} m V^2$$

$$= \frac{1}{2} m_{pellet} \cdot \left(2\sqrt{\frac{GM_{\oplus}}{r}}\right)^2$$

$$= \frac{1}{2}\, 0.004 \cdot 4 \cdot \frac{6.6726\times10^{-11} \cdot 5.98\times10^{24}}{(6.37\times10^{6} + 500\times10^{3})}$$

← $R_{\oplus} + 500km$

$$= \underline{4.646538341\times10^{5} J}$$

(b)
$$\frac{KE_{\text{in space}}}{KE_{\text{rifle}}} = \frac{4.64\times10^{5} J}{\frac{1}{2} m_{bullet} \cdot V^2}$$

$$= \frac{4.64\cdots\times10^{5} J}{1.805\times10^{3} J}$$

$$= \underline{2.574259468\times10^{2}}$$