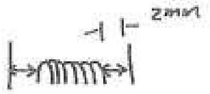
ch 15, #1, 11, 37, 43, 44, 72, 96, 102

#1



$\nu = 120\ Hz$

$\omega = 2\pi\nu$

(a) 2mm back & forth → Amplitude = $\underline{1\,mm}$

(b)
$$x = A\sin(\omega t + \phi) \longrightarrow ①$$
$$\dot{x} = A\omega\cos(\omega t + \phi) \longrightarrow ②$$
$$\ddot{x} = -A\omega \sin(\omega t + \phi) \longrightarrow ③$$

when $\ddot{x} = 0$, $\dot{x}$ is max

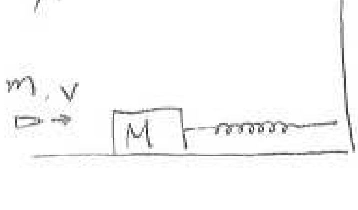
Eqn. ③
$$\ddot{x} = -A\omega \sin(\omega t + \phi) = 0$$
$$\omega t + \phi = n\pi \quad (n \text{ is an integer}) \longrightarrow ③'$$

② ← ③'
$$\dot{x} = A\omega \cos(n\pi)$$
$$= (0.001)\cdot(2\pi\cdot 120)\cdot 1$$
$$= \underline{0.7539822369\ m/sec}$$

(c) max $\ddot{x}$ is when $\dddot{x} = 0$
$$\dddot{x} = -A\omega^3 \cos(\omega t + \phi) = 0$$
$$\rightarrow \cos(\omega t + \phi) = 0$$
$$\therefore (\omega t + \phi) = n\frac{\pi}{2} \quad (n \text{ is an integer})$$
$$|\ddot{x}| = |-A\omega^2 \sin(n\tfrac{\pi}{2})|$$
$$= (0.001)\cdot(2\pi\cdot 120)^2\cdot(1)$$
$$= \underline{5.684892135 \times 10^2\ m/sec^2}$$

#11. See with the additional problem #3 on this chapter.

#37.



(a) cons. of P

p_i		p_f
mV	=	$(m+M)V_f$

$$\therefore V_f = \left(\frac{m}{m+M}\right)V$$

(b) KE upon the impact.

$$\frac{1}{2}(m+M)V_f^2 = \frac{1}{2}(m+M)\left[\left(\frac{m}{m+M}\right)V\right]^2$$
$$= \frac{1}{2}\frac{m^2}{(m+M)}V^2$$

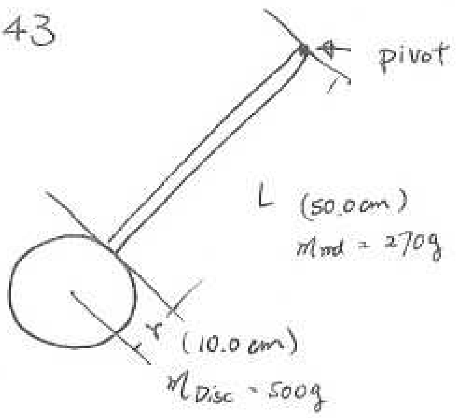
compression of the spring (E conserves)

E_i | E_f
KE. | S.E.

$$\frac{1}{2}\frac{m^2}{(m+M)}V^2 = \frac{1}{2}kX^2$$

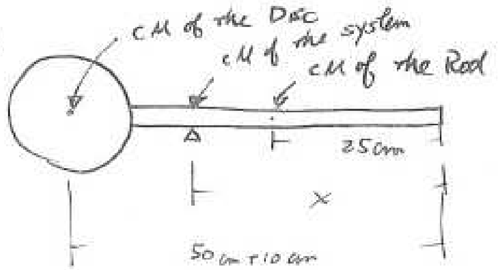
$$\therefore X = \sqrt{\frac{m^2V^2}{k(m+M)}} = \frac{mV}{\sqrt{k(m+M)}}$$

#43



(a)
$$I = I_{rod} + I_{Disc}$$
$$= \frac{1}{3}M_{rod}L_{rod}^2 + \left(\frac{1}{2}M_{Disc}R_{Disc}^2 + M_{Disc}(L_{rod}+R_{Disc})^2\right)$$
$$= \frac{1}{3}(0.270\,kg)(0.5m)^2 + \left(\frac{1}{2}(0.5kg)(0.1m)^2 + (0.5kg)(0.5m+0.1m)^2\right)$$
$$= 0.205\ kgm^2$$

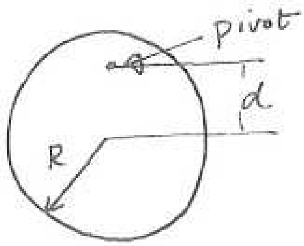
(b)



$$x\,(M_{Disc}+M_{Rod}) = 60cm\,(M_{Disc}) + 25cm\,(M_{rod})$$
$$x\,(500g+270g) = 60\,cm\,(500g) + 25cm\,(270g)$$
$$x = 47.72\ cm$$

(c)
$$\omega = \sqrt{\frac{M_T\,g\,d}{I}} \qquad (d = x)$$
$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{M_T\,g\,d}} = 2\pi\sqrt{\frac{0.205}{(0.27+0.5)\,981\cdot 0.4772}}$$
$$= 1.498282212\ sec$$

#44



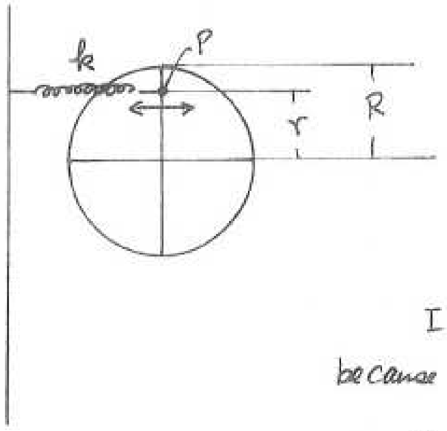
$$\omega = \sqrt{\frac{Mgd}{I}}$$

$$I = I_{CM} + I_{Disc} = Md^2 + \frac{1}{2}MR^2 = M\left(d^2 + \frac{1}{2}R^2\right)$$

$$\omega = \sqrt{\frac{Mgd}{I}} = \sqrt{\frac{Mgd}{M(d^2+\frac{1}{2}R^2)}} = \sqrt{\frac{gd}{d^2+\frac{1}{2}R^2}}$$

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^2+\frac{1}{2}R^2}{gd}} = 2\pi\sqrt{\frac{(0.0175)^2 + \frac{1}{2}(0.0235)^2}{9.81 \cdot 0.0175}} = \underline{\underline{0.365955014 \text{ sec}}}$$

#72



By the spring force, there is a τ at P

$$\tau = \vec{r} \times \vec{F} = \vec{r} \times (-kx) = I\alpha$$

$$I\alpha + rkx = 0$$

because the oscillation is small $x \sim r\theta$

$$\therefore I\alpha + kr^2\theta = 0$$

$$I\ddot{\theta} + kr^2\theta = 0$$

Let $\theta = A\cos(\omega t + \phi)$

$$\dot{\theta} = -A\omega\sin(\omega t + \phi)$$

$$\ddot{\theta} = -A\omega^2\cos(\omega t + \phi)$$

$$\therefore I\left(-A\omega^2\cos(\omega t+\phi)\right) + kr^2\left(A\cos(\omega t+\phi)\right) = 0$$

$$A\left(-I\omega^2 + kr^2\right)\cos(\omega t+\phi) = 0$$

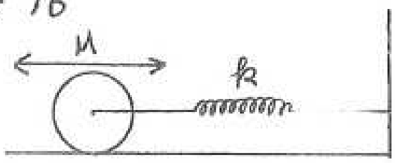
$$\therefore I\omega^2 + kr^2 = 0$$

(a) $\omega = \sqrt{\frac{kr^2}{I}} = \sqrt{\frac{kr^2}{MR^2}} = \frac{r}{R}\sqrt{\frac{k}{M}}$

(b) $r = R \qquad \omega = \sqrt{\frac{k}{M}}$

(c) $r = 0 \qquad \omega = 0$

\# 96



(a) At the equilibrium

$$E_i = E_f$$

$$SE = KE + RE$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 \quad \text{—— ①}$$

$$I_{cylinder} = \frac{1}{2}MR^2 \quad \text{—— ②}$$

$$V = \omega R \quad \text{so} \quad \omega = \frac{V}{R} \quad \text{—— ③}$$

① ⟵ ② & ③

$$\frac{1}{2}kx^2 = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{V}{R}\right)^2$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mV^2 + \frac{1}{4}mV^2$$

when you look at this eqn. you can recognize SE will be change to 2 : 1 ratio of KE : RE.

(a) $KE = \frac{2}{3} SE = \frac{2}{3}\left(\frac{1}{2}kx^2\right) = \frac{2}{3}\left(\frac{1}{2}\cdot 3\cdot(0.25)^2\right) = \underline{\underline{0.0625\,J}}$

(b) $R.E. = \frac{1}{3} SE = \frac{1}{3}\left(\frac{1}{2}kx^2\right) = \underline{\underline{0.03125\,J}}$

(c) $E_{Total} = SE + KE + RE = \text{Constant}$

$$\frac{1}{2}kx^2 + \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = \text{const.}$$

see above

$$\frac{1}{2}kx^2 + \frac{1}{2}mV^2 + \frac{1}{4}mV^2 = \text{const}$$

$$\frac{1}{2}kx^2 + \frac{3}{4}mV^2 = \text{const.}$$

Here is a greate step.

$$\frac{dE_{total}}{dt} = \frac{d\left(\frac{1}{2}kx^2 + \frac{3}{4}mV^2\right)}{dt} = \frac{d\,(\text{const})}{dt} = 0\,!$$

$$\frac{1}{2}\,2kx\cdot\frac{dx}{dt}+\frac{3}{4}\,2mV\cdot\frac{dv}{dt}=0$$

$$kxV+\frac{3}{2}mV\cdot a=0$$

$$V\left(kx+\frac{3}{2}ma\right)=0$$ (since V is not always zero.)

$$kx+\frac{3}{2}ma=0$$

$$kx+\frac{3}{2}m\ddot{x}=0$$

Let

$$x=A\cos(\omega t+\phi)$$
$$\dot{x}=-A\omega\quad(\omega t+\phi)$$
$$\ddot{x}=-A\omega^2\cos(\omega t+\phi)$$

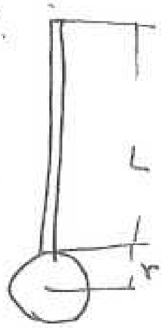
$$kA\cos(\omega t+\phi)+\frac{3}{2}m\left(-A\omega^2\cos(\omega t+\phi)\right)=0$$

$$A\cos(\omega t+\phi)\left(k-\frac{3}{2}m\omega^2\right)=0$$

$$\therefore\quad \omega=\sqrt{\frac{2k}{3M}}$$

$$\therefore\quad P=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{3M}{2k}}$$

102.



$r=0.15\,m$
$P=2.0\,sec$
$g=9.800\,m/sec^2$

$$I=(L+r)^2m+\frac{1}{2}mr^2$$

$$\omega=\sqrt{\frac{mgd}{I}}\quad \& \quad P=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{I}{mgd}}$$

$$\therefore\quad P=2\,sec=2\pi\sqrt{\frac{m(L+r)^2+\frac{1}{2}mr^2}{mg(L+r)}}$$

$$\vdots$$

$$\pi L^2+(2\pi r-g)L+\left(\frac{3}{2}\pi^2r^2-gr\right)=0$$

$$\therefore\quad L=0.831485381\,m$$

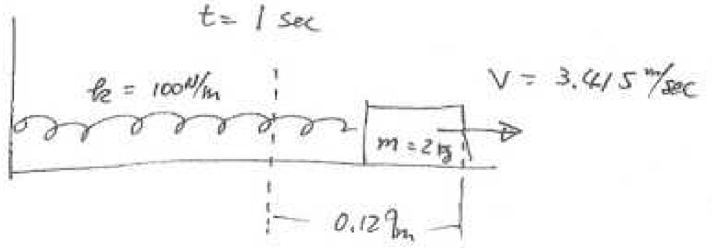
Ch. 15 additional problems:

#1 A simple harmonic oscillator consists of a block of mass 2.00 kg attached to a spring of spring constant 100 N/m. What t = 1.00 sec, the position and velocity of the block are x = 0.129 m and v = 3.415 m/sec. (a) What is the amplitude of the oscillations? What were the (b) position and (c) velocity of the block at t = 0 sec?

#2 A massless spring hangs from the ceiling with a small object attached to its lower end. The object is initially held at rest in a position y_i and oscillates up and down, with its lowest position being 10 cm below y_i. (a) What is the frequency of the oscillation? (b) What is the speed of the object when it is 8.0 cm below the initial position? (c) An object of mass 300 g is attached to the first object, after which the system oscillates with half the original frequency. What is the mass of the first object? (d) Relative to y_i, where is the new equilibrium (rest) position with both objects attached to the spring?

#3 Suppose that the two spring in fig 15-30 have different spring constant k_1 and k_2. Show that the frequency f of oscillation of the block is then given by $\sqrt{\nu_1^2 + \nu_2^2}$

#1



$$\left.\begin{aligned} X &= A\cos(\omega t+\phi) \\ \dot{x} &= -A\omega \sin(\omega t+\phi) \\ \ddot{x} &= -A\omega^2\cos(\omega t+\phi) \end{aligned}\right\} \quad t = 1\,\text{sec} \rightarrow \left\{\begin{aligned} X &= A\cos(\omega+\phi) = 0.129 \quad \text{—①} \\ \dot{x} &= -A\omega\sin(\omega+\phi) = 3.415 \quad \text{—②} \\ \ddot{x} &= -A\omega^2\cos(\omega+\phi) \end{aligned}\right.$$

So,

Eqn. ①. $0.129 = A\cos\left(\sqrt{\frac{k}{m}} + \phi\right) \qquad \left(\omega = \sqrt{\frac{k}{m}}\right)$

Eqn ② $3.415 = -A\sqrt{\frac{k}{m}}\sin\left(\sqrt{\frac{k}{m}}+\phi\right) \rightarrow 3.415\sqrt{\frac{m}{k}} = -A\sin\left(\sqrt{\frac{k}{m}}+\phi\right)$ —②'

$①^2 + ②^2$

a) $$A^2\cos^2\left(\sqrt{\frac{k}{m}}+\phi\right) + A^2\sin^2\left(\sqrt{\frac{k}{m}}+\phi\right) = (0.129)^2 + \left(3.415\sqrt{\frac{m}{k}}\right)^2$$

$$A^2(\overbrace{\cos^2 + \sin^2}^{1}) = (0.129)^2 + \left(3.415\sqrt{\frac{2}{100}}\right)^2$$

$$A^2 = (0.129)^2 + \left(3.415\sqrt{\frac{2}{100}}\right)^2$$

$$\therefore \ \underline{\underline{A = 0.4998854869\ m}}$$

(b) First we need to calculate ϕ

$$\tan(\omega t+\phi) = \frac{\sin(\omega+\phi)}{\cos(\omega+\phi)} = \frac{②'}{①'} = \frac{-3.415\sqrt{\frac{m}{k}}}{0.129}$$

[to calculate an angle, it is always a good idea to use $\tan^{-1}$ because it tells us which quad. we are looking at. For example, in this case, if only eqn. ① was used, we would think the ans. was +. However, by looking at both sin & cos., we know the ans. is in the 4th quad.]

$$\sqrt{\frac{k}{m}} + \phi = \tan^{-1}\frac{-3.415\sqrt{\frac{2}{100}}}{0.129}$$

$$= -1.309783607 \text{ rad}$$

$$\therefore \phi = -8.380851419 \text{ rad}$$

$$x = A\cos(\omega t+\phi)$$

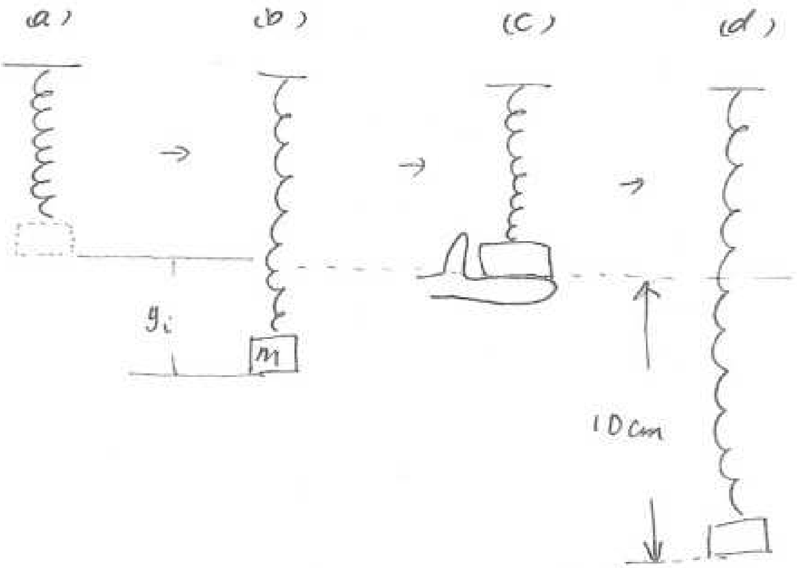
$$= 0.498854869\cos(\omega t - 8.380851419)$$

$$= \underline{\underline{-0.251357467 4\, m}}$$

(c) $\dot{x} = -A\sqrt{\frac{k}{m}}\sin(\omega t - 8.38085 1419)$

$$= \underline{\underline{3.055363503 \text{ m/sec}}}$$

#2



(a) In the textbook, they use f as frequency, but I will use ν (nu) for frequency.

$$m = \frac{1}{4}(m+0.3)$$

$$m - \frac{1}{4}m = \frac{0.3}{4}$$

$$\frac{3}{4}m = \frac{0.3}{4}$$

$$m = \frac{0.3}{3} = \underline{\underline{0.1 \text{ kg}}}$$

(d) $$F = -kx = -mg$$

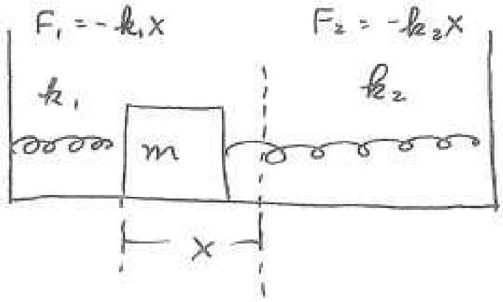
$$k = \frac{mg}{x} = \frac{0.1 \cdot 9.81}{0.05} = \frac{98.1}{5}$$

$$F_{new} = -k X_{new} = -m_{new}\, g$$

$$X = \frac{(0.1+0.3)\,9.81}{\frac{98.1}{5}}$$

$$= \frac{5 \cdot 0.4 \cdot 9.81}{98.1} = \underline{\underline{0.2 \text{ m}}}$$

11 # 3



$$F_{net} = F_1 + F_2 = -(k_1 x - k_2 x) = -(k_1 + k_2)x$$

$$ma = -(k_1 + k_2)x$$

$$m\ddot{x} + (k_1 + k_2)x = 0$$

Let

$$x = A\cos(\omega t + \phi)$$

$$\ddot{x} = -A\omega^2 \cos(\omega t + \phi)$$

$$m\left(-A\omega^2 \cos(\omega t + \phi)\right) + (k_1 + k_2)A\cos(\omega t + \phi) = 0$$

$$A\cos(\omega t + \phi)\left(-m\omega^2 + (k_1 + k_2)\right) = 0$$

$$m\omega^2 = k_1 + k_2$$

$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

How can we find ω (thus frequency) w/o any k or m?
The spring stretches when a mass is suspended from it (fig.(b))
because there is a force (weight of m in this case) acting
on the spring.

$$F = -k y_i = -mg \qquad y_i \text{ is a half of a displacement} = 0.05m$$

$$\Rightarrow \frac{k}{m} = \frac{g}{y_i}$$

$$\underbrace{\omega = \sqrt{\frac{k}{m}}}_{\text{You should be able to derive this.}} = \sqrt{\frac{g}{y_i}} = \sqrt{\frac{9.81}{0.05}} = \underline{\underline{2.229365734 \text{ Hz}}}$$

(b)

$$x = 0.05 \cos(\omega t + \phi) \quad \text{——— ①}$$

$$\dot{x} = -0.05\,\omega \sin(\omega t + \phi)$$

$$= -0.05\sqrt{\frac{9.81}{0.05}} \sin(\omega t + \phi) \quad \text{——— ②}$$

eqn ①

$$x = 8 - 5\text{ cm} = 0.03\text{ m} = 0.05 \cos(\omega t + \phi)$$

$$\therefore \frac{0.03}{0.05} = \cos(\omega t + \phi)$$

$$\therefore \cos^{-1}\frac{0.03}{0.05} = (\omega t + \phi) \quad \text{——— ①}'$$

$$② \leftarrow ①'$$

$$|\dot{x}| = \left| -0.05\sqrt{\frac{9.81}{0.05}} \sin\left(\cos^{-1}\frac{0.03}{0.05}\right) \right| = \underline{\underline{0.5602856414 \text{ m/sec}}}$$

(c)

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{y_i}} = \sqrt{\frac{9.81}{0.05}} \Rightarrow \nu_0 = \frac{\omega_0}{2\pi}$$

$$\omega_0 = \sqrt{\frac{k}{m+0.3}} \Longrightarrow \nu = \frac{\omega}{2\pi}$$

$$\frac{\nu}{\nu_0} = \frac{\frac{\omega}{2\pi}}{\frac{\omega_0}{2\pi}} = \frac{\omega}{\omega_0} = \frac{\sqrt{\frac{k}{m+0.3}}}{\sqrt{\frac{k}{m}}} = \sqrt{\frac{m}{m+0.3}} = \frac{1}{2}$$

$$\frac{m}{m+0.3} = \left(\frac{1}{2}\right)^2$$

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \quad (\text{if } k_1 = k_2)$$

modefying $\omega = \sqrt{\frac{k}{m}}$

$$\omega_1 = \sqrt{\frac{k_1}{m}} \quad \& \quad \omega_2 = \sqrt{\frac{k_2}{m}}$$

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \quad \& \quad \nu_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}}$$

$$\therefore \frac{k_1}{m} = 4\pi^2\nu_1^2 \quad \& \quad \frac{k_2}{m} = 4\pi\nu_2^2$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1}{m} + \frac{k_2}{m}}$$

$$= \frac{1}{2\pi}\sqrt{4\pi^2\nu_1^2 + 4\pi\nu_2^2}$$

$$= \frac{1}{2\pi}\sqrt{4\pi^2(\nu_1^2 + \nu_2^2)} = \underline{\underline{\sqrt{\nu_1^2 + \nu_2^2}}}$$