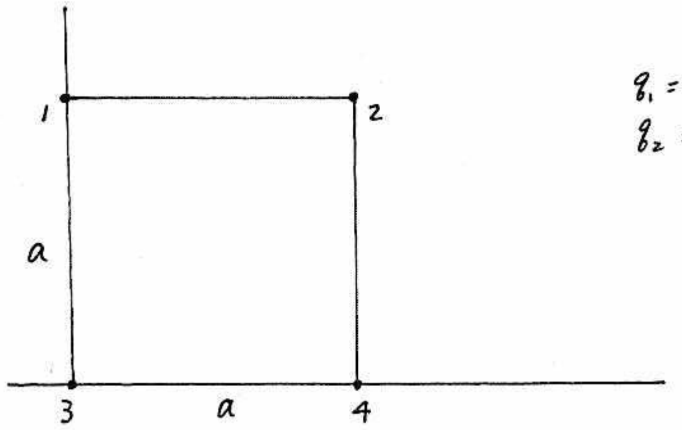
Ch 21. # 6, 20, 25, 65, 66

#6.



$q_1 = q_4 = Q$
$q_2 = q_3 = q$

(a) $\Sigma \vec{F}_{on1} = \vec{F}_{2on1} + \vec{F}_{3on1} + \vec{F}_{4on1} = 0$ (Given condition)

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\hat{j} + \frac{1}{4\pi\varepsilon}\frac{Q^2}{(\sqrt{2}a)^2}\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$= \frac{1}{4\pi\varepsilon}\frac{Q}{a^2}\left(q + \frac{Q}{2\sqrt{2}}\right)\hat{i} + \frac{1}{4\pi\varepsilon}\frac{Q}{a^2}\left(q + \frac{Q}{2\sqrt{2}}\right)\hat{j} = 0\hat{i} + 0\hat{j}$$

$\therefore\ q + \frac{Q}{2\sqrt{2}} = 0$

$\therefore\ \frac{Q}{q} = -2\sqrt{2} = \underline{\underline{-2.82842712 5}}$ (they are opposite charges)

$\Sigma \vec{F}_{on3} = \vec{F}_{1on3} + \vec{F}_{2on3} + \vec{F}_{4on3} = 0$

$$= \frac{1}{4\pi\varepsilon}\frac{Qq}{a^2}\hat{j} + \frac{1}{4\pi\varepsilon}\frac{q^2}{(\sqrt{2}a)^2}\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) + \frac{1}{4\pi\varepsilon}\frac{Qq}{a^2}\hat{i} = 0\hat{i} + 0\hat{j}$$

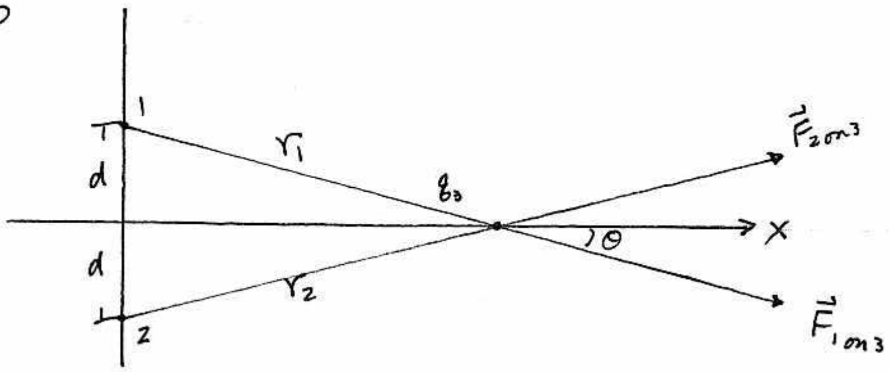
$$= \frac{1}{4\pi\varepsilon}\frac{q}{a^2}\left(Q + \frac{q}{2\sqrt{2}}\right)\hat{i} + \frac{1}{4\pi\varepsilon}\frac{q}{a^2}\left(Q + \frac{q}{2\sqrt{2}}\right)\hat{j}$$

$\therefore\ Q + \frac{q}{2\sqrt{2}} = 0$

$\therefore\ \frac{Q}{q} = -\frac{1}{2\sqrt{2}} = \underline{\underline{-0.35355339}}$

(b) There is no way to satisfy both conditions at the same time.

#20



$q_1 = q_2 = 3.20\times10^{-19}$ C

$q_3 = 6.40\times10^{-19}$ C $(= 2q_1)$

$d = 0.17$ m

$0\text{ cm} \le x \le 5$ m

DO NOT PLUG IN NUMBERS UNTIL THE END !

Because $q_1 = q_2$ and $r_1 = r_2$, $|F_{1on3}| = |F_{2on3}|$.

Also, because of the symmetry $\Sigma \vec{F}_y = 0$. therefore $\Sigma \vec{F}_x = 2|F_{1on3,x}|$

$$\Sigma \vec{F}_x = 2\cdot\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_1^2}\cos\theta \qquad \cos\theta = \frac{x}{r_1} \ \ \& \ \ r_1 = (d^2+x^2)^{1/2}$$

$$= 2\cdot\frac{1}{4\pi\varepsilon_0}\frac{q_1(2q_1)}{r_1^2}\cdot\frac{x}{r_1}$$

$$= \frac{4q_1^2}{4\pi\varepsilon_0}\cdot\frac{x}{r_1^3} = \underline{\frac{q_1^2}{\pi\varepsilon_0}\frac{x}{(d^2+x^2)^{3/2}}} \qquad \text{①}$$

this is a typical maximization / Minimization problem.

$$\frac{d\vec{F}_x}{dx} = \frac{d\left[\frac{q_1^2}{\pi\varepsilon_0}\frac{x}{(d^2+x^2)^{3/2}}\right]}{dx} = \frac{q_1^2}{\pi\varepsilon_0}\left(\frac{1}{(d^2+x^2)^{3/2}} - \frac{3}{2}x(d^2+x^2)^{-5/2}\cdot 2x\right)$$

$$= \frac{q_1^2}{\pi\varepsilon_0}\left(\frac{1}{(d^2+x^2)^{3/2}} - \frac{3x^2}{(d^2+x^2)^{5/2}}\right)$$

$$= \frac{q_1^2}{\pi\varepsilon_0}\left(\frac{d^2+x^2-3x^2}{(d^2+x^2)^{5/2}}\right)$$

$$= \frac{q_1^2}{\pi\varepsilon_0}\left(\frac{d^2-2x^2}{(d^2+x^2)^{5/2}}\right) = 0$$

$$\therefore\ d^2 - 2x^2 = 0$$

$$\therefore\ \underline{\underline{x = \frac{d}{\sqrt{2}}}}$$

(a) & (c) Minimum is at $\underline{\underline{x = 0}}$ (Forces from q_1 & q_2 are equal & opposite $\therefore \underline{\underline{\Sigma \vec{F} = 0}}$)

(b) & (d) Max. is at $x = \frac{d}{\sqrt{2}} = \underline{\underline{0.120208152\ m}}$ —— ②

① ← ②

$$\Sigma \vec{F}_x = \frac{q_1^2}{\pi \varepsilon_0} \frac{x}{(d^2 + x^2)^{3/2}}$$

$$= \frac{q_1^2}{\pi \varepsilon_0} \cdot \frac{d/\sqrt{2}}{\left(d^2 + \left(\frac{d}{\sqrt{2}}\right)^2\right)^{3/2}}$$

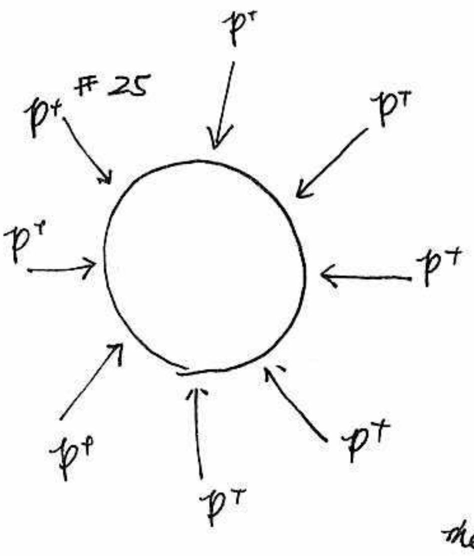
$$= \frac{q_1^2}{\pi \varepsilon_0} \frac{d/\sqrt{2}}{\left(d^2 + \frac{d^2}{2}\right)^{3/2}}$$

$$= \frac{q_1^2}{\pi \varepsilon_0} \frac{d/\sqrt{2}}{\left(\frac{3d^2}{2}\right)^{3/2}}$$

$$= \frac{q_1^2}{\pi \varepsilon_0} \frac{d/\sqrt{2}}{\frac{3^{3/2}}{2^{3/2}} \cdot d^3} = \frac{q_1^2}{\pi \varepsilon_0} \cdot \frac{2}{3^{3/2} \cdot d^2}$$

$$= \frac{(3.20 \times 10^{-19} C)^2}{\pi \cdot 8.85 \times 10^{-12}} \cdot \frac{2}{\sqrt{27} \cdot (0.17)^2} = \underline{\underline{4.90520417 \times 10^{-26} N}}$$

#25



rate: $1500\ p^+/m^2/sec$

Total surface area of the earth $= 4\pi R_\oplus^2$

$R_\oplus = 6.37 \times 10^6 m$

Hence the total # of protons hitting the earth:

$1500\ p^+/m^2/sec \cdot 4\pi R_\oplus^2 = 7.648565457 \times 10^{17}\ p^+/sec$

$= \frac{dq}{dt}$

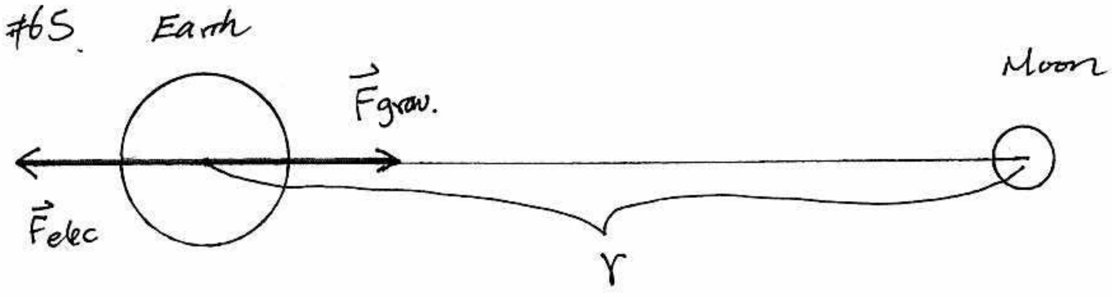
the earth is getting 7.65×10^{17} protons per sec. means this is the rate of change of charge ($\frac{dq}{dt}$) which is "current". We have to change the charge into coulombs

$7.648565457 \times 10^{17}\ p^+/sec \times \frac{1.6 \times 10^{-19} C}{1\ p^+}$

$= 1.22 \times 10^{-1}\ C/sec$

$= \underline{\underline{0.122\ amp}}$

#65.



$$F_{grav.} = G\frac{m_\oplus m_{☾}}{r^2}$$

$$F_{elec} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad (q_1 = q_2 \text{ : Given condition})$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1^2}{r^2}$$

(a)

$$\Sigma\vec{F} = G\frac{M_\oplus M_{☾}}{r^2} - \frac{1}{4\pi\varepsilon}\frac{q_1^2}{r^2} = 0$$

Solve for q_1

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1^2}{r^2} = G\frac{M_\oplus M_{☾}}{r^2}$$

$$\therefore q_1 = \sqrt{4\pi\varepsilon_0 G M_\oplus M_{☾}}$$

$$= \sqrt{4\cdot\pi\cdot 8.85\times10^{-12}\cdot 6.6726\times10^{-11}\cdot 5.98\times10^{24}_{kg}\cdot 7.36\times10^{22}_{kg}}$$

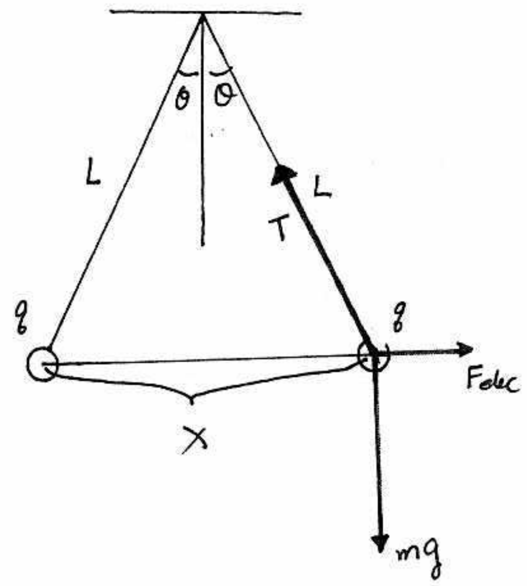
$$= \underline{\underline{5.714965488\times10^{13}\ C}}$$

(b) Both ($F_{grav.}$ & $F_{elec.}$) are ruled by Inverse-Square-Law so when they are set equal to each other, r^2 is cancelled.

(c) $1\ p^+ = 1.6\times10^{-19}\ C$, $1.673\times10^{-27}\ kg$

$$5.7149\ldots\times10^{13}\ C\cdot\frac{1.673\times10^{-27}\ kg/\text{proton}}{1.6\times10^{-19}\ C/\text{proton}} = \underline{\underline{5.975710788\times10^{5}\ kg}}$$

#66. Just because you are in Phy. 231 does not mean you can forget what you learned in Phy. 230. You will encounter lots of problems that require the methods you learned in 230. This is one of them.



Steps (Just in case of you forgot)

1. Draw a diagram
2. Pick a point
3. Draw a vector / vectors applied to the point
4. Break it / them into x - y components.
5. Write eqns (for both x & y) about the point

(a) In this case, there are three forces (see the diagram) and $\Sigma \vec{F} = 0$

X-comp

$$F_{elec} - T\sin\Theta = 0$$

$$\frac{1}{4\pi\varepsilon}\frac{q^2}{x^2} - T\sin\Theta = 0$$

$$\therefore T\sin\Theta = \frac{1}{4\pi\varepsilon}\frac{q^2}{x^2} \quad \text{—— ①}$$

y-comp

$$T\cos\Theta - mg = 0$$

$$\therefore T\cos\Theta = mg \quad \text{—— ②}$$

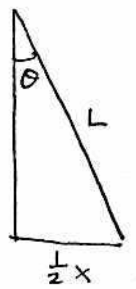
$$\frac{①}{②}$$

$$\frac{T\sin\Theta}{T\cos\Theta} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}}{mg}$$

$$\tan\Theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$

$$\wr$$

$$\sin\Theta \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$



$$\sin\Theta = \frac{\frac{1}{2}x}{L}$$

$$\frac{\frac{1}{2}x}{L} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$

$$x^3 = \frac{1}{4\pi\varepsilon_0}\frac{2Lq^2}{mg}$$

$$\therefore x = \left(\frac{L}{2\pi\varepsilon_0}\frac{q^2}{mg}\right)^{1/3}$$

(b) Solve for q : $q = \pm\sqrt{\dfrac{x^3 \cdot 2\pi\varepsilon_0 mg}{L}}$

$$\left\{\begin{aligned} x &= 5\,cm = 5\times10^{-2}\,m \\ L &= 120\,cm = 1.2\,m \\ m &= 10\,g = 1\times10^{-2}\,kg \end{aligned}\right\}$$ Do Not Forget to convert them into SI units!!

$$q = \pm\sqrt{\frac{(5\times10^{-2})^3 \cdot 2\cdot\pi\,\varepsilon_0\,(1\times10^{-2})(9.81)}{1.2}}$$

$$= \pm\ 2.384\times10^{-8}\,C$$