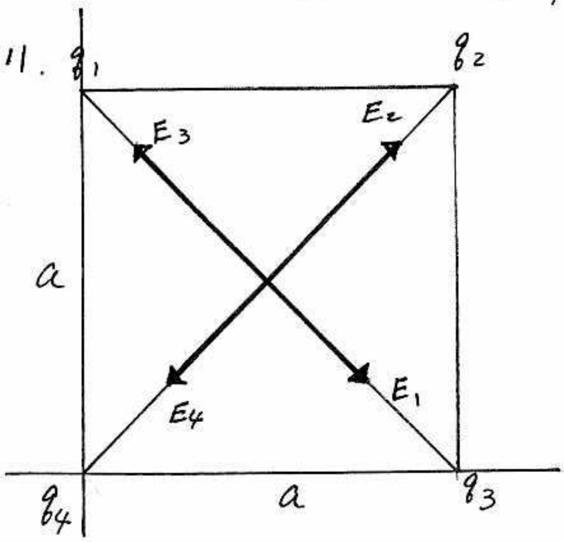
Ch.22 # 11, 27, 29, 55, 84, 86

\# 11.



$a = 5\,cm$

$q_1 = +10\,nC$

$q_2 = -20\,nC$

$q_3 = 20\,nC$

$q_4 = -10\,nC$

(As you can see $\Sigma \vec{E}_x = 0$ .. only noticable when you draw a diagram very accurate)

$\therefore$ $\vec{E}_{Total}$ is up.

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} - \frac{q_1}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

[treating all the charges as positive since we know their directions]

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} + \frac{q_2}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon}\left(-\frac{q_3}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} + \frac{q_3}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\vec{E}_4 = \frac{1}{4\pi\varepsilon}\left(-\frac{q_4}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} - \frac{q_4}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\Sigma\vec{E} = \frac{1}{4\pi\varepsilon_0}\Bigg[\left(\frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} - \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} - \frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\frac{1}{\sqrt{2}}\right)\hat{i}$$
$$+\left(-\frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} - \frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\right)\hat{j}\Bigg]$$

$$= \frac{1}{4\pi\varepsilon_0}\left(0\hat{i} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$= \underline{\underline{0\hat{i} + 1.017306572\times10^{5}\,N/C\,\hat{j}}}$$

\# 27.

(a) $\lambda = \frac{Q}{L} = \frac{-4.23\times10^{-15}C}{0.0815m} = -5.190184049\times10^{-14}\,C/m$

For the calculations of E fields for non-point charged objects, use the <u>same</u> <u>steps</u> <u>Do Not forget these steps</u> practice!

<u>step 1</u>

Draw a diagram (of course). Take a small piece of a charged object (dq). Draw an E field line from it at the point.

<u>step 2</u>

Write an eqn. of dE created by dq at the point.

<u>step 3</u>

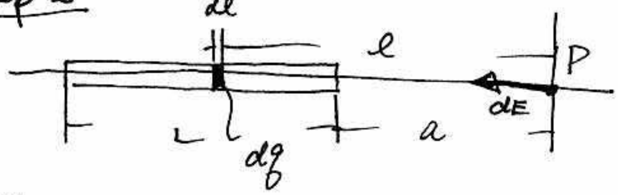
check symmetry to see if E_x or E_y is cancelled.

<u>step 4</u>

Integrate.

(b)

<u>step 1</u>



dE is toward the rod because the charge is negative.

<u>step 2</u>

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{l^2} \qquad dq = \lambda \cdot dl$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot dl}{l^2}$$

<u>step 3</u>

In this case, there is no "y" direction.

<u>step 4</u>

$$E = \int dE = \int_a^{a+L} \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot dl}{l^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left. -\frac{1}{l}\right|_a^{a+L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{L}{a(a+L)}\right)$$

$(= 1.573008017 \times 10^{-3}$ N/C w/ values given $)$

(c) As noted in (b), the direction of the E field is toward the rod

(d) w/ $a = 50m$ $\quad E = 1.518937486 \times 10^{-8}$ N/C

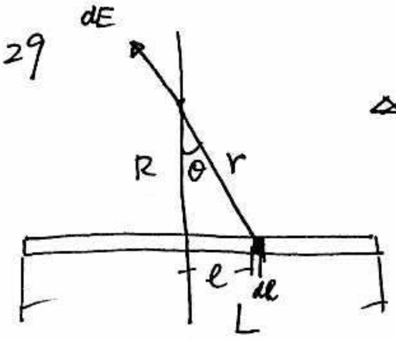
(e) $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = 1.521413354 \times 10^{-8}$ N/C

The questions are trying to tell you that when the solution is

$\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{L}{a(a+L)}\right)$ and $a >>>>> L$, the result is $\sim \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a^2}$

(remember $\lambda = \frac{q}{L}$ and $\lambda L = q$. so you can treat as a point charge)

#29



Step 1

Step 2

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot dl}{(l^2+R^2)} \qquad \begin{cases} \lambda = \frac{Q}{L} \\ r = (R^2+l^2)^{1/2} \end{cases}$$

Step 3

$E_x = 0$ by symmetry (Don't forget to write this even if this is obvious to you!)

Step 4

$$dE_y = dE\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+R^2)} \cdot \frac{R}{(R^2+l^2)^{1/2}} \qquad \left(\cos\theta = \frac{R}{r}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\, dl}{(l^2+R^2)^{3/2}}$$

Since there is no "l" in the numerator $\longrightarrow$ Trig sub

Let $\tan\theta = \frac{l}{R}$

$$\begin{cases} l = R\tan\theta \\ dl = R\sec^2\theta \cdot d\theta \end{cases}$$

So

$$dE_y = \frac{\lambda}{4\pi\varepsilon_0}\frac{R\,(R\sec^2\theta \cdot d\theta)}{(R^2\tan^2\theta + R^2)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{R^2\sec^2\theta \cdot d\theta}{R^3(\tan^2\theta+1)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{R}\frac{\sec^2\theta \cdot d\theta}{\sec^3\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R} \cdot \frac{1}{\sec\theta} \cdot d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta \cdot d\theta$$

$$E_y = \int dE_y = \int_{\theta_{min}}^{\theta_{max}} \frac{\lambda}{4\pi\varepsilon R} \cos\theta \cdot d\theta$$

$$= 2\int_0^{\theta_{max}} \frac{\lambda}{4\pi\varepsilon R} \cos\theta \cdot d\theta$$

$$= \frac{2\lambda}{4\pi\varepsilon R} \sin\theta \Big|_0^{\theta_{max}}$$

$$= \frac{2\lambda}{4\pi\varepsilon R} \frac{l}{(R^2+l^2)^{1/2}} \Big|_0^{l=\frac{1}{2}L}$$

$$= \frac{2\lambda}{4\pi\varepsilon R} \left(\frac{\frac{1}{2}L}{(R^2+(\frac{1}{2}L)^2)^{1/2}}\right) = \frac{\lambda}{2\pi\varepsilon_0 R} \frac{\frac{1}{2}L}{(R^2+\frac{1}{4}L^2)^{1/2}}$$

$$= \frac{\lambda}{2\pi\varepsilon R} \cdot \frac{\frac{1}{2}L}{\frac{1}{2}(4R^2+L^2)^{1/2}} = \underline{\underline{\frac{\lambda}{2\pi\varepsilon_0 R} \frac{L}{(4R^2+L^2)^{1/2}}}}$$

$$= \frac{\frac{Q}{L}}{2\pi\varepsilon R} \frac{L}{(4R^2+L^2)^{1/2}}$$

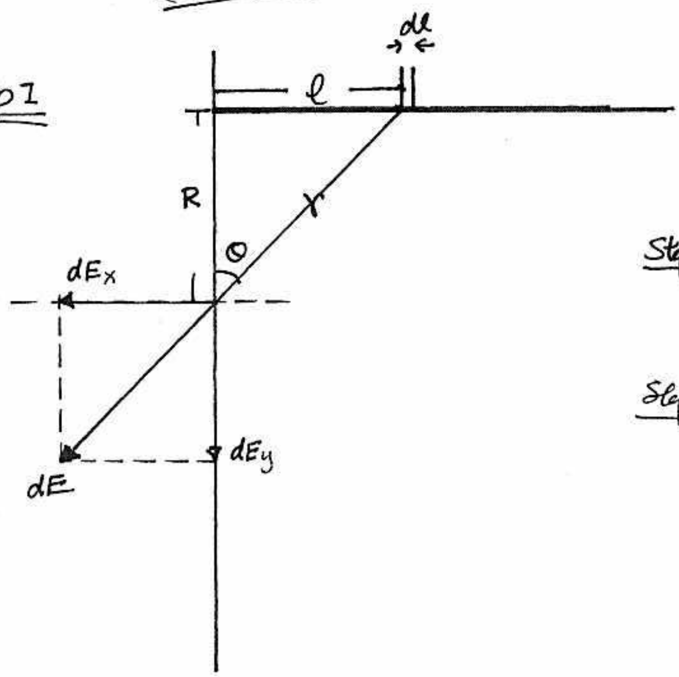
$$= \underline{\underline{\frac{Q}{2\pi\varepsilon R} \frac{1}{(4R^2+L^2)^{1/2}}}}$$

(a) $E_y = \frac{7.81\times10^{-9}}{2\pi\varepsilon_0 (0.145m)} \cdot \frac{1}{(4(0.06)^2+(0.145)^2)^{1/2}} = \underline{\underline{2.7343\,24373\times10^{4}\ N/C}}$

(b) $\underline{\underline{+y \text{ direction}}}$

#55

Step I



Step 2

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2}$$

Step 3

In this case there's no symmetry ∴

X comp

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta$$ (Not worrying about the direction)

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+R^2)}\cdot\frac{l}{(l^2+R^2)^{1/2}}$$

y comp

$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+R^2)}\cdot\frac{R}{(l^2+R^2)^{1/2}}$$

step 4

$$E_x = \int dE_x = \int\frac{\lambda}{4\pi\varepsilon_0}\frac{l\, dl}{(l^2+R^2)^{3/2}}$$ (u-sub)

Let $\begin{cases} u = l^2+R^2 \\ du = 2l\, dl \end{cases}$

$$= \int\frac{\lambda}{4\pi\varepsilon_0}\frac{1}{2}\frac{2l\,dl}{(l^2+R^2)^{3/2}}$$

$$= \frac{\lambda}{8\pi\varepsilon_0}\int\frac{du}{u^{3/2}}$$

$$= \frac{\lambda}{8\pi\varepsilon_0}\; -2\frac{1}{u^{1/2}}$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{u^{1/2}}$$

$$= \frac{-\lambda}{4\pi\varepsilon_0}\frac{1}{(l^2+R^2)^{1/2}}\Bigg|_0^{\infty}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{R}$$

$$E_y = \int dE_y = \int\frac{\lambda R}{4\pi\varepsilon_0}\frac{dl}{(l^2+R^2)^{3/2}}$$ (Trig-sub)

Let $\tan\theta = \frac{l}{R}$

$\begin{cases} l = R\tan\theta \\ dl = R\sec^2\theta\cdot d\theta \end{cases}$

$$= \frac{\lambda R}{4\pi\varepsilon_0}\int\frac{R\sec^2\theta\cdot d\theta}{(R^2\tan^2\theta+R^2)^{3/2}}$$

$$= \frac{\lambda R^2}{4\pi\varepsilon_0}\int\frac{\sec^2\theta\cdot d\theta}{R^3(\tan^2\theta+1)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}\cdot\int\frac{\sec^2\theta\cdot d\theta}{\sec^3\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}\int\frac{1}{\sec\theta}\cdot d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}\int\cos\theta\cdot d\theta$$

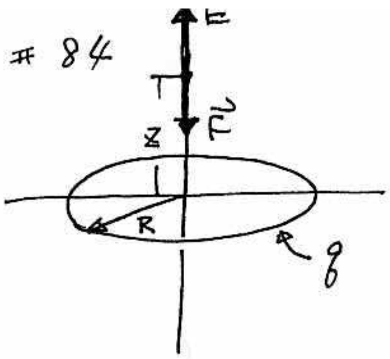
$$= \frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Bigg|_0^{\pi/2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}$$

$$\theta_{final} = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1)$$

$$= \underline{\underline{45^\circ}}$$

\# 84



$$E = \frac{qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$$

$$F = -Ee^- = m_e a_z$$

Because $\vec{F}$ & $\vec{E}$ are opposite to each other

$$\therefore -\frac{qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \cdot e^- = m_e a_z$$

$$-\frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \cdot z = m_e \ddot{z} \qquad \left(\ddot{z} = \frac{d^2 z}{dt^2} = a_z\right)$$

$$\frac{qe^-}{4\pi\varepsilon (z^2+R^2)^{3/2}} z + m_e \ddot{z} = 0$$ — you should be able to recognize this is a siple harmonic oscillation!

Let $z = A\cos(\omega t + \phi)$

$\dot{z} = -A\omega \sin(\omega t + \phi)$

$\ddot{z} = -A\omega^2 \cos(\omega t + \phi)$

$$\frac{qe^-}{4\pi\varepsilon (z^2+R^2)^{3/2}} A\cos(\omega t+\phi) - m_e A\omega^2 \cos(\omega t + \phi) = 0$$

$$\left(\frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} - m_e\omega^2\right)(A\cos(\omega t+\phi)) = 0$$

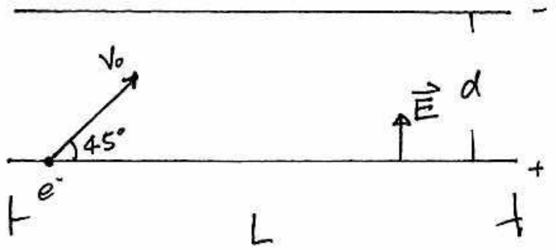
$$\therefore \frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} - m_e\omega^2 = 0$$

$$\therefore \quad \omega = \sqrt{\frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2} m_e}}$$

for a small displacement ($R >> z$)

$$\omega = \sqrt{\frac{qe^-}{4\pi\varepsilon_0 m_e R^3}}$$

86



$V_0 = 6 \times 10^6$ m/sec

$E = 2 \times 10^3$ N/C

$d = 2$ cm $= 0.02$ m

$L = 10$ cm $= 0.1$ m

x comp

$a_x = 0$ ——— ①

$V_x = V_0 \cos 45°$ ——— ②

$X = V_0 t \cos 45°$ $(\int V_x \cdot dt)$ ——— ③

y comp

$F_{elec} = -Ee^-$ (F on the electron is down)

We can ignore $F_{grav.}$ because $F_{elec} >> F_{grav.}$

$F_{elec} = -Ee^- = m_e a_y$ ——— ④

$\therefore a_y = -\frac{Ee^-}{m_e}$

$V_y = \int a_y \cdot dt = -\frac{Ee^-}{m_e}t + \cancel{V_{0y}}$ ——— ⑤ ($V_0 \sin 45°$)

$y = \int V_y \cdot dt = -\frac{Ee^-}{2m_e}t^2 + V_0 t \sin 45° + \cancel{y_0}$ ——— ⑥ (0)

Don't be scare by all these letters.
This is another application of Physics 230. We are using; F = ma, a, V, & X or y relations. In this case, the applied force is electrical, not gravitational – that is the only difference.

(a) Let's check to see if the e^- will hit the top plate
Eqn. ⑥. set $y = 2 \times 10^{-2}$ m, solve for t

$$2 \times 10^{-2} = -\frac{Ee^-}{2m_e}t^2 + V_0 t \sin 45°$$

$$\frac{Ee^-}{2m_e}t^2 - V_0 t \cancel{\sin 45°}^{\frac{1}{\sqrt{2}}} + 2 \times 10^{-2} = 0$$

$$t = \frac{\frac{V_0}{\sqrt{2}} \pm \sqrt{\left(\frac{V_0}{\sqrt{2}}\right)^2 - 4\left(\frac{Ee^-}{2m_e} \cdot 2 \times 10^{-2}\right)}}{2 \cdot \frac{Ee^-}{2m_e}}$$

$$= \frac{4.24 \times 10^{6} \pm 1.9873 \times 10^{6}}{3.5126 \times 10^{14}}$$

$$= 6.421 \times 10^{-9} \text{ sec or } 1.77 \times 10^{-8} \text{ sec}$$

the second solution is as if there is no top plate

& the E field is constant (above the space of the top plate)

In reality, the electron will hit the top plate at 6.421×10^{-9} sec

→ Yes, it does hit the top plate (at $t = 6.421 \times 10^{-9}$ sec — ⑥′)

(b) ③ ← ⑥′

$$X = V_0 t \cos 45^\circ$$

$$= 6. \times 10^6 \cdot t \cdot \frac{1}{\sqrt{2}}$$

$$= 2.724 \times 10^{-2} \, m$$