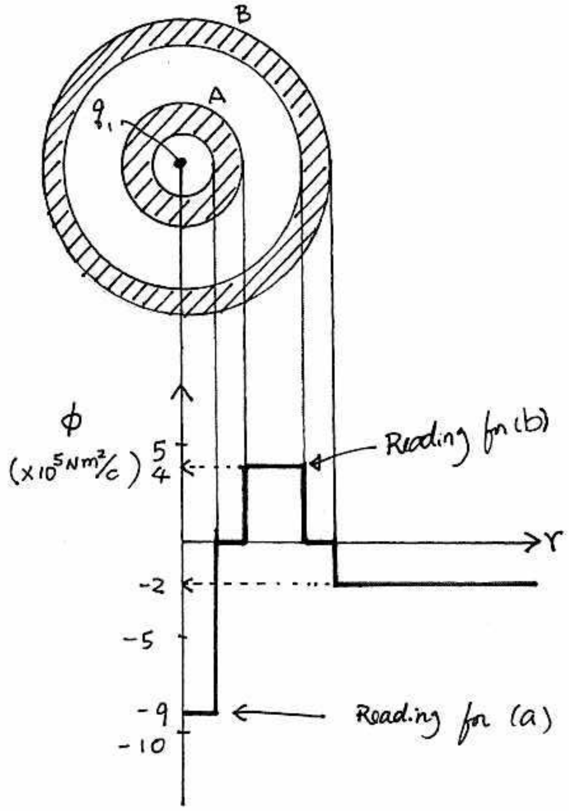
Ch 23. #18, 26, 29, 30, 39, 47, 48, 50, 53, extra 83

#18



(a) $\Phi_E = \oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$= E \cdot 4\pi r^2 = \frac{q_1}{\varepsilon_0} = -9 \times 10^5 \text{ Nm}^2/\text{C}$

$\therefore q_1 = \varepsilon_0 (-9 \times 10^5 \text{ Nm}^2/\text{C}) = \underline{\underline{-7.965 \times 10^{-6} \text{ C}}}$

(b) $\Phi_E = \oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$= E \cdot 4\pi r^2 = \frac{q_1 + q_A}{\varepsilon_0} = 4 \times 10^5 \text{ Nm}^2/\text{C}$

$\therefore q_A = \varepsilon_0 (4 \times 10^5 \text{ Nm}^2/\text{C}) - q_1$

$= \varepsilon_0 (4 \times 10^5 \text{ Nm}^2/\text{C}) - \varepsilon_0 (-9 \times 10^5 \text{ Nm}^2/\text{C})$

$= \varepsilon_0 (13 \times 10^5 \text{ Nm}^2/\text{C}) = \underline{\underline{1.1505 \times 10^{-5} \text{ C}}}$

(c) $\Phi_E = \oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

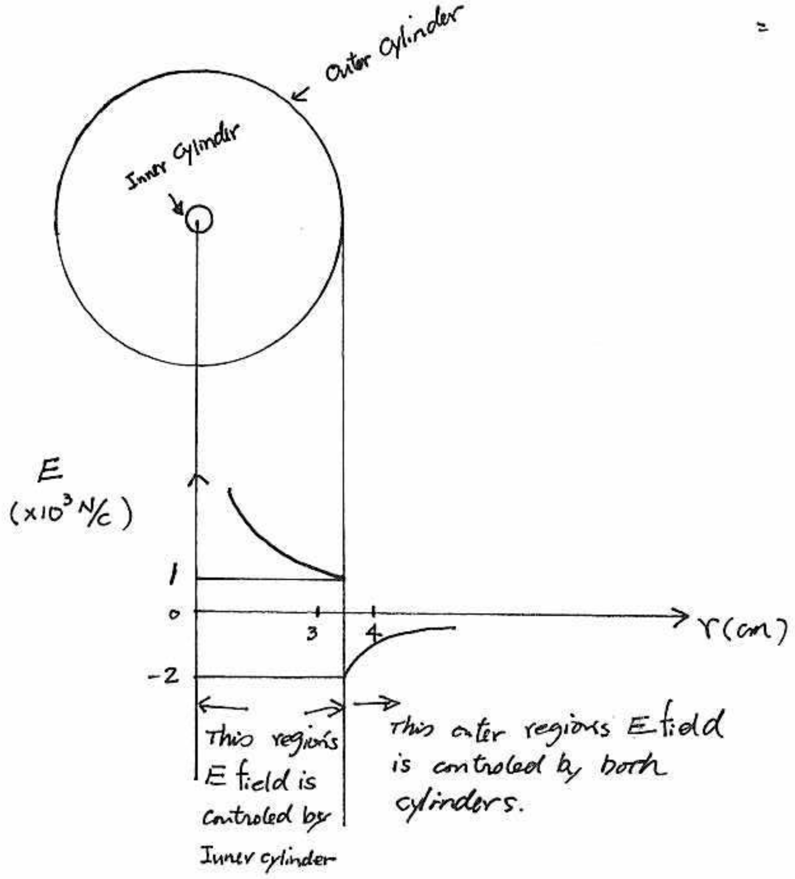
$= E \cdot 4\pi r^2 = \frac{q_1 + q_A + q_B}{\varepsilon_0} = -2 \times 10^5 \text{ Nm}^2/\text{C}$

$q_B = \varepsilon_0 (-2 \times 10^5 \text{ Nm}^2/\text{C}) - (q_1 + q_A)$

$= \varepsilon_0 (-2 \times 10^5 \text{ Nm}^2/\text{C}) - \varepsilon_0 (4 \times 10^5 \text{ Nm}^2/\text{C})$

$= \varepsilon_0 (-6 \times 10^5 \text{ Nm}^2/\text{C}) = \underline{\underline{-5.31 \times 10^{6} \text{ C}}}$

#28



$R_{\text{inner cylinder}} < r < R_{\text{outer cylinder}}$

$$\oint E_1 \, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_1 \, 2\pi r L = \frac{\lambda_{inner} \cdot L}{\varepsilon_0}$$

$$\lambda_{inner} = E_1 (2\pi r) \varepsilon_0 \qquad (E_1 = 1\times10^3 \, N/C \text{ at } 3.5\,cm\ (0.035m)) \text{ — ①}$$

$R_{\text{outer cylinder}} < r$

$$\oint E_2 \, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_2 \cdot 2\pi r L = \frac{(\lambda_{inner} + \lambda_{outer})L}{\varepsilon_0}$$

$$\lambda_{outer} = E_2 (2\pi r) \varepsilon_0 - \lambda_{inner} \qquad (E_2 = -2\times10^3 \, N/C \text{ at } 0.035m)$$

Sub. eqn ①

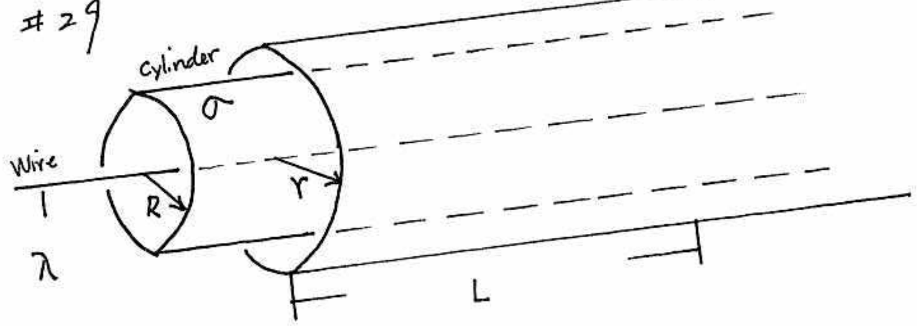
$$= E_2 (2\pi r) \varepsilon_0 - E_1 (2\pi r) \varepsilon_0$$

$$= (E_2 - E_1)(2\pi r) \varepsilon_0$$

$$= (-2\times10^3 \, N/C - 1\times10^3 \, N/C)(2\pi (0.035m))\, 8.85\times10^{-12}$$

$$= -5.838649947\times10^{-9} \, C/m$$

#29



For $r > R$, $E = 0$

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

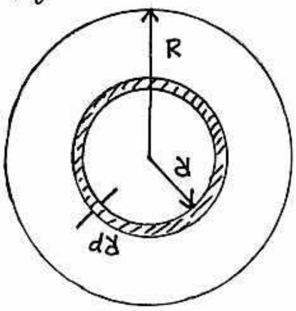
$$E \cdot 2\pi r \cdot L = \frac{\lambda L + \sigma(2\pi R)L}{\varepsilon_0}$$

$$E = \frac{\lambda + \sigma(2\pi R)}{2\pi r \varepsilon_0} = 0$$

$$\therefore \lambda + \sigma(2\pi R) = 0$$

$$\sigma = \frac{-\lambda}{2\pi R} = \frac{-(-3.6\times10^{-9} \, C/m)}{2\pi (1.5\times10^{-2} m)} = 3.8197\times10^{-8} \, C/m^2$$

#38



$$dq = \rho(2\pi\varrho \cdot d\varrho)L \quad (L \text{ for cylinder})$$

$$= A\varrho^2(2\pi\varrho\, d\varrho)L$$

$$= A\, 2\pi L\, \varrho^3 \cdot d\varrho$$

$$\therefore q \,(\text{from the center to } r < R) = \int dq$$

$$= \int_0^r A\, 2\pi L\, \varrho^3 \cdot d\varrho$$

$$= A\, 2\pi L\, \frac{1}{4}\varrho^4 \Big|_0^r$$

$$= \frac{A\pi L r^4}{2}$$

(a) for $r < R$ ($r = 0.03\,m$, $R = 0.04\,m$ in this case)

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\, 2\pi r L = \frac{\int dq}{\varepsilon_0}$$ → see above for evaluation of q

$$E\, 2\pi r L = \frac{A\pi L r^4}{2\varepsilon_0}$$

$$\therefore E = \frac{Ar^3}{4\varepsilon_0} = \frac{2.5\times10^{-6}\,\text{C/m}^2 \cdot (0.03)^3}{4\ \varepsilon_0} = \underline{1.906779661\ \text{N/C}}$$

(b) for $r > R$ ($r = 0.05\,m$, $R = 0.04\,m$)

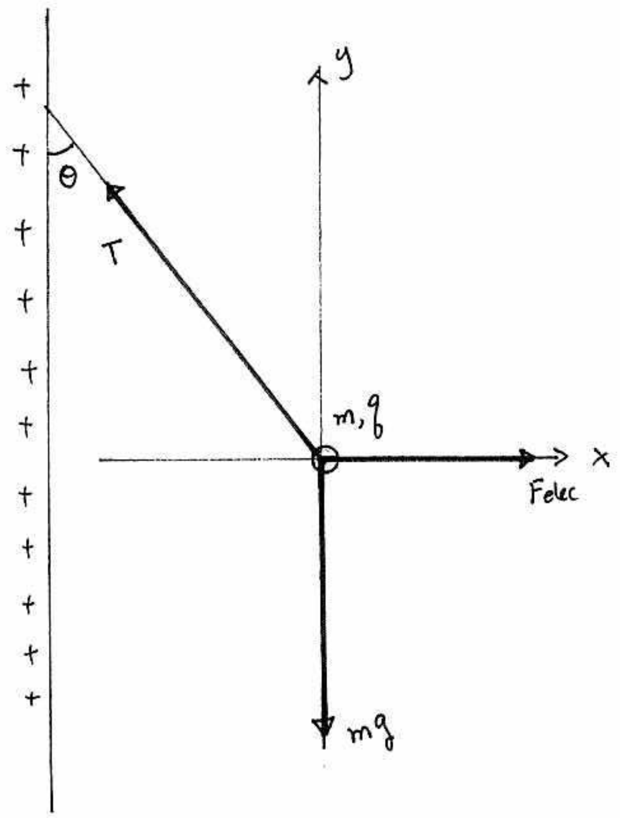
$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r L = \frac{\int_0^R dq}{\varepsilon_0}$$ → Gaussian Surface is outside the cylinder. We need to integrate from 0 to the surface of the cylinder

$$E \cdot 2\pi r L = \frac{A\pi L R^4}{2\varepsilon_0}$$

$$\therefore E = \frac{AR^4}{4r\varepsilon_0} = \frac{2.5\times10^{-6}\ (0.04)^4}{4\ (0.05)\ \varepsilon_0} = \underline{3.615819209\ \text{N/C}}$$

#39



$m = 1.0\ mg = 1.0 \times 10^{-6}\ kg$

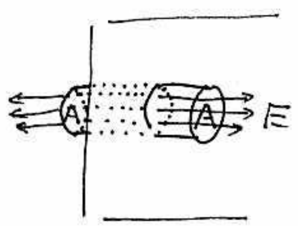
$q = 2 \times 10^{-8}\ C$

$\theta = 30°$

$F_{elec} = qE$

$\Sigma \vec{F} = \vec{T} + \vec{mg} + \vec{F}_{elec} = 0.$

First, $\vec{E}$ for a large sheet



$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$E \cdot 2A = \frac{\sigma A}{\varepsilon_0}$

$\therefore\ E = \frac{\sigma}{2\varepsilon_0}$ — ①

Second, Analysis of Force.

X comp

$F_{elec} - T \sin 30° = 0$

$T \sin 30° = F_{elec}$ — ②

y comp

$T \cos 30° - mg = 0$

$T \cos 30° = mg$ — ③

$\frac{②}{③} \leftarrow ①$

$$\frac{T \sin 30°}{T \cos 30°} = \frac{F_{elec}}{mg}$$

$$\tan 30° = \frac{qE}{mg} = \frac{q \frac{\sigma}{2\varepsilon_0}}{mg}$$

solve for σ

$$\sigma = \frac{mg \tan 30°}{q\, 2\varepsilon_0} = \frac{(1 \times 10^{-6} kg)(9.81\ m/sec^2) \tan 30°}{(2 \times 10^{-8} C) \cdot 2 \cdot 8.85 \times 10^{-12}} = 5.012468435 \times 10^{-9}\ C/m^2$$

#47

Sorry about the bad drawing

q_r in the shell from a up to $r (< b)$

Take a thin shell with radius, γ and a thickness $d\gamma$

$dq = \rho \cdot d(\text{vol})$ ← of a thin shell

$= \rho \cdot (4\pi \gamma^2 \cdot d\gamma)$ $\left(\rho = \frac{A}{\gamma} : \text{Given}\right)$

$= \frac{A}{\gamma}(4\pi\gamma^2 \cdot d\gamma)$

$= 4\pi A \gamma \cdot d\gamma$

$q_r = \int dq = \int_a^r 4\pi A \gamma \cdot d\gamma$

$= 2\pi A (r^2 - a^2)$ — ① (charge in the shell up to r)

Gauss' Law

$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$E \cdot 4\pi r^2 = \frac{q_1 + q(\text{in the shell up to } r)}{\varepsilon_0}$ — ②

② ← ①

$E = \frac{q_1 + 2\pi A(r^2 - a^2)}{4\pi\varepsilon_0 r^2}$

The other condition given to this problem is that E is <u>constant</u> in the shell $(a \le r \le b)$ ⇒ $\frac{dE}{dr} = 0$

$\frac{dE}{dr} = \frac{d\left(\frac{q_1 + 2\pi A(r^2 - a^2)}{4\pi\varepsilon_0 r^2}\right)}{dr} = \frac{d\left(\frac{q_1}{4\pi\varepsilon_0 r^2} + \frac{A}{2\varepsilon_0} - \frac{Aa^2}{2\varepsilon_0 r^2}\right)}{dr}$

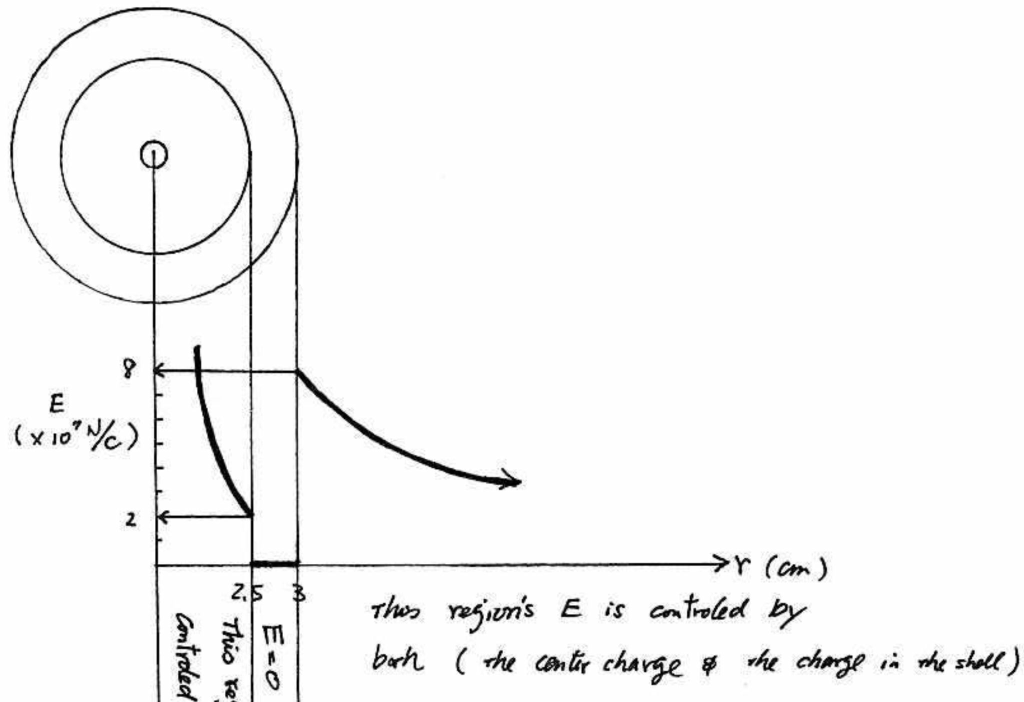
$= \frac{-2q_1}{4\pi\varepsilon_0 r^3} - \frac{-2Aa^2}{2\varepsilon_0 r^3}$

$= \frac{-q_1 + 2\pi A a^2}{2\pi\varepsilon_0 r^3} = 0$

$\therefore -q_1 + 2\pi A a^2 = 0$

$A = \frac{q_1}{2\pi a^2} = \frac{45\,fC}{2\pi(0.02\,m)^2} = \frac{45\times10^{-15}\,C}{2\pi(0.02\,m)^2} = \underline{1.79049311\times10^{-11}\,C/m^2}$

\# 48



<u>Case 1</u> $(0 < r_1 < 2.5\,cm)$

$$\oint E_1 \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_1\, 4\pi r_1^2 = \frac{q_1}{\varepsilon_0}$$

$$q_1 = E_1\, 4\pi r_1^2\, \varepsilon_0 \quad (\text{at } r_1 = 2.5\,cm,\ E_1 = 2\times10^7\,N/C) \qquad ①$$

<u>Case 2</u> $(r_2 > 2.5\,cm)$

$$\oint E_2 \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_2 \cdot 4\pi r_2^2 = \frac{q_1 + q_{shell}}{\varepsilon_0} \qquad ②$$

② ← ①

$$E_2 \cdot 4\pi r_2^2 = \frac{E_1\, 4\pi r_1^2 \varepsilon_0 + q_{shell}}{\varepsilon_0}$$

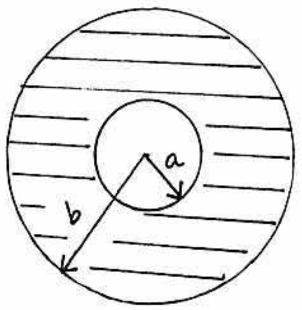
$$\therefore\ q_{shell} = E_2\, 4\pi r_2^2 \varepsilon_0 - E_1\, 4\pi r_1^2 \varepsilon_0 \quad (\text{at } r_2 = 3.0\,cm,\ E_2 = 8\times10^7\,N/C)$$

$$= (E_2 r_2^2 - E_1 r_1^2)\, 4\pi \varepsilon_0$$

$$= \left[8\times10^7\,N/C\,(0.03\,m)^2 - 2\times10^7\,N/C\,(0.025\,m)^2\right] 4\pi \varepsilon_0$$

$$= \underline{\underline{6.617136606\times10^{-6}\,C}}$$

#50



$\rho = 1.84\ nC/m^3 = 1.84 \times 10^{-9}\ C/m^3$

$a = 10.0\ cm = 0.1\ m$

$b = 2a = 20.0\ cm = 0.2\ m$

(a) $r = 0$

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$\therefore \underline{\underline{E = 0}}$ (because there is no - charge inside)

(b) $r = \frac{a}{2} = 0.05m$

$$E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$\therefore \underline{\underline{E = 0}}$ (still there is no charge inside)

(c) $r = a$

$$E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$\therefore \underline{\underline{E = 0}}$

(d) $r = 1.5a$

$$E \cdot 4\pi r^2 = \frac{\int_a^{1.5a} \rho \cdot 4\pi r^2 \cdot dr}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{4\pi\rho}{3\varepsilon_0}\left[(1.5a)^3 - (a)^3\right]$$

$$\therefore E = \frac{\rho}{3\varepsilon_0}\ \frac{(1.5a)^3 - (a)^3}{(1.5a)^2} = \underline{\underline{7.315337937\ N/C}}$$

(e) $r = 2a$ (use the same eqn. used in (d))

$$E = \frac{\rho}{3\varepsilon_0}\ \frac{(2a)^3 - (a)^3}{(2a)^2} = \underline{\underline{12.12806026\ N/C}}$$

(f) $r = 3b$

$$E \cdot 4\pi r^2 = \frac{\int_a^b \rho\, 4\pi r^2 \cdot dr}{\varepsilon_0} = \frac{4\pi\rho}{3\varepsilon_0}(b^3 - a^3)$$

$$\therefore E = \frac{\rho}{3\varepsilon_0 (3b)^2}(b^3 - a^3) = \underline{\underline{1.347562252\ N/C}}$$

#53

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \; 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$\therefore \; E = \frac{q_{enc}}{4\pi \varepsilon r^2} = K r^2 \quad \text{(Given condition)}$$

$$\therefore \; q_{enc} = 4\pi \varepsilon_0 K r^6 \quad \text{——— ①}$$

Also from the given condition,

"A charge distribution is spherically symmetric" — within a thin shell, the charge density is uniform.

$$\Rightarrow \; dq_{(\text{thin shell})} = \rho \cdot 4\pi \partial^2 \cdot d\partial$$

"... but not uniform radially" — so ρ is a variable charge density and is a fcn of radius

$$\Rightarrow \; \rho \propto \partial^n$$

(nth power since we do not know if ρ is proportional to ∂^1, ∂^2, ∂^3, or what)

$\therefore$ Let $\rho = \text{ꟿ} \partial^n$ where ꟿ is some constant (could use "K" because "K" is already taken.)

$$\therefore \; dq = \text{ꟿ} \partial^n \cdot 4\pi \partial^2 \cdot d\partial = \text{ꟿ} 4\pi \partial^{n+2} \cdot d\partial$$

$$\therefore \; q_{enc} = \int_0^r \text{ꟿ} 4\pi \partial^{n+2} \cdot d\partial$$

$$= \frac{\text{ꟿ} 4\pi}{n+3} r^{n+3} \quad \text{——— ②}$$

① = ②

$$q_{enc} = 4\pi \varepsilon K r^6 = \text{ꟿ} \frac{4\pi}{n+3} r^{n+3}$$

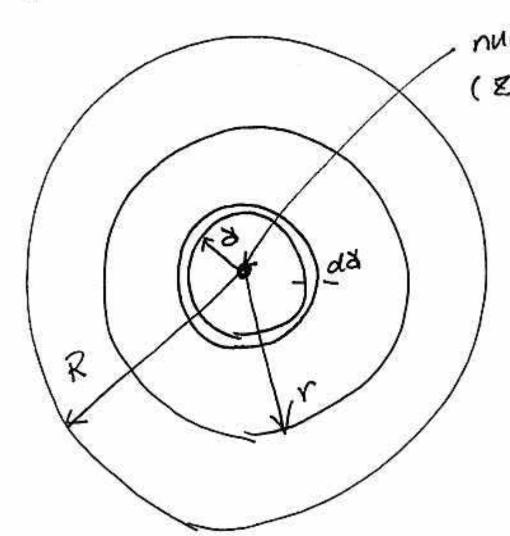
$$\therefore \; n = 3$$

$$\varepsilon_0 K = \frac{\text{ꟿ}}{3+3}$$

$$\text{ꟿ} = 6\varepsilon_0 K$$

$$\therefore \; \underline{\rho = 6\varepsilon_0 K r^3}$$

83



$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

charge of the nucleus; charge of electron cloud up to 'r'.

$$E \cdot 4\pi r^2 = \frac{Ze + \int_0^r \rho \cdot 4\pi \eth^2 \cdot d\eth}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Ze + \rho \frac{4\pi r^3}{3}}{\varepsilon_0}$$

$$\rho = \frac{\text{total charge}}{\text{Vol}} = \frac{-Ze}{\frac{4}{3}\pi R^3}$$

$$\therefore \quad E \cdot 4\pi r^2 = \frac{Ze + \left(\frac{-Ze}{\frac{4}{3}\pi R^3} \cdot \frac{4\pi r^3}{3}\right)}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Ze - Ze\frac{r^3}{R^3}}{\varepsilon_0}$$

$$\underline{\underline{E = \frac{Ze}{4\pi\varepsilon} \left(\frac{1}{r^2} - \frac{r}{R^3}\right)}}$$

Extra

$$\rho = \frac{\rho_s \eth}{R}$$

(a) dq (of the shell) $= \rho \cdot 4\pi \eth^2 \cdot d\eth$

$$= \frac{\rho_s \eth}{R} 4\pi \eth^2 \cdot d\eth$$

$$= \rho_s \frac{4\pi \eth^3}{R} \cdot d\eth$$

$$\therefore \quad Q = \int dq = \int_0^R \rho_s \frac{4\pi\eth^3}{R} \cdot d\eth = \frac{4\pi\rho_s}{R} \cdot \frac{1}{4} \eth^4 \Big|_0^R = \frac{\pi\rho_s R^4}{R}$$

$$= \underline{\underline{\pi \rho_s R^3}}$$

(b) Hence $\rho_s = \frac{Q}{\pi R^3}$

Gauss' Law

$$\oint E \cdot dA = \frac{q_{enc.}}{\varepsilon_0}$$

$$\varepsilon_0 E \cdot 4\pi r^2 = \int_0^r \rho \cdot 4\pi \eth^2 \cdot d\eth = \int_0^r \frac{\rho_s \eth}{R} \cdot 4\pi \eth^2 \cdot d\eth = \int_0^r \frac{\rho_s}{R} 4\pi \eth^3 \cdot d\eth$$

$$\varepsilon_0 E \, 4\pi r^2 = \frac{4\pi\rho_s}{R} \cdot \frac{1}{4} \eth^3 \Big|_0^r = \frac{\pi\rho_s}{R} r^4$$

$$\therefore \quad E = \frac{\pi \rho_s r^4}{4\pi\varepsilon_0 R r^2} = \frac{\frac{Q}{\pi R^3} r^2}{4\varepsilon_0 R}$$

$$= \underline{\frac{Q}{4\pi\varepsilon_0} \frac{r^2}{R^4}}$$