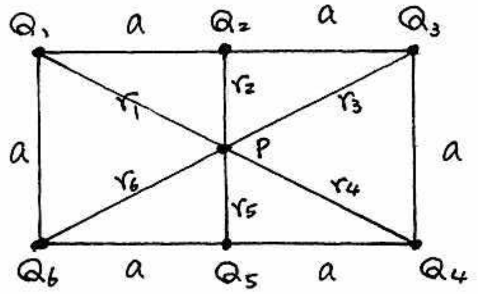
ch 24 # 16, 21, 28, 29, 34, 35, 36, 37, 104, 114

#16



$Q_1 = Q_4 = +2q_1$

$Q_2 = Q_5 = +4q_2$

$Q_3 = -3q_1$

$Q_6 = -q_1$

$r_1 = r_3 = r_4 = r_6 = \sqrt{3}\,a$

$r_2 = r_5 = \frac{1}{2}a$

$$V = \Sigma V_i = \Sigma \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

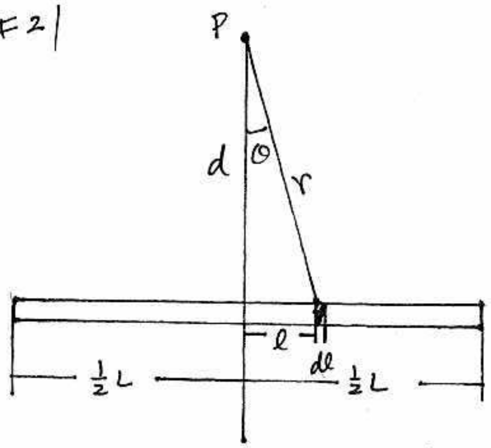
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \frac{Q_4}{r_4} + \frac{Q_5}{r_5} + \frac{Q_6}{r_6}\right)$$

$$= \frac{1}{4\pi\varepsilon}\left[\left(\frac{Q_1 + Q_3 + Q_4 + Q_6}{\sqrt{3}\,a}\right) + \left(\frac{Q_2 + Q_5}{\frac{1}{2}a}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{+2q_1 - 3q_1 + 2q_1 - q_1}{\sqrt{3}\,a}\right) + \left(\frac{4q_2 + 4q_2}{\frac{1}{2}a}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{16q_2}{a} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{16\cdot 6\times 10^{-12}C}{0.39m} = \underline{\underline{2.213367309 \text{ volts}}}$$

#21



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$dq = \lambda\, dl$

$r = (l^2 + d^2)^{1/2}$

$$\therefore\ dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+d^2)^{1/2}}$$

(a)

$$V = \int dV = \int_{-\frac{1}{2}L}^{\frac{1}{2}L}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+d^2)^{1/2}} = 2\int_0^{\frac{1}{2}L}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+d^2)^{1/2}}$$

trig sub

Let $\tan\theta = \frac{l}{d}$

$l = d\tan\theta$

$dl = d\sec^2\theta\cdot d\theta$

$$\therefore\ V = 2\int_0^{\frac{1}{2}L}\frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\, d\sec^2\theta}{(d^2\tan^2\theta + d^2)^{1/2}} = \frac{2\lambda}{4\pi\varepsilon_0}\int_0^{\frac{1}{2}L}\frac{d\sec^2\theta\cdot d\theta}{d\sec\theta}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\frac{1}{2}L}\sec\theta\cdot d\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \ln|\sec\theta+\tan\theta| = \frac{\lambda}{2\pi\varepsilon_0} \ln\left|\frac{r}{d}+\frac{\ell}{d}\right| = \frac{\lambda}{2\pi\varepsilon_0} \ln\left|\frac{(\ell^2+d^2)^{1/2}+\ell}{d}\right| \Bigg|_0^{\frac{1}{2}L}$$

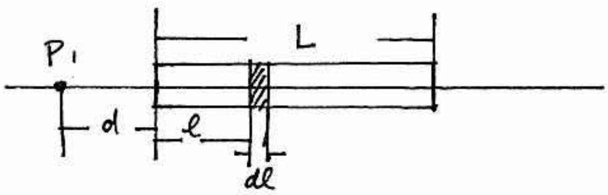
$$= \frac{3.68\times10^{-12}\ C/m}{2\pi\ \varepsilon_0} \ln\left|\frac{((\frac{1}{2}L)^2+d^2)^{1/2}+\frac{1}{2}L}{d}\right| \Bigg|_0^{\frac{1}{2}L}$$

$L =$ cm
$d =$
$\lambda = 3.68\ pC/m$

$$= \underline{\underline{2.4269717\times10^{-2}\ V}}$$

(b) $V = 0$ $\quad |V_{by(+)}| = |V_{by(-)}|$

#28



$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cdot d\ell}{(d+\ell)}$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, d\ell}{(d+\ell)}$$

$$= \frac{\lambda}{4\pi\varepsilon} \ln(d+\ell)\Big|_0^L$$

$$= \underline{\underline{\frac{\lambda}{4\pi\varepsilon_0} \ln\frac{(d+\ell)}{d}}}$$

$Q = 56.1\ fC = 56.1\times10^{-15}\ C$
$L = 0.12m$
$d = 0.025m$

plus in the values given

$$= \underline{7.389452298\times10^{-3}\ \text{volts}}$$

#29 using x instead of ℓ

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dX}{(X+d)} \quad \text{where } \lambda = cx$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{cx\, dx}{(X+d)}$$

$$= \frac{c}{4\pi\varepsilon_0}\frac{x}{(x+d)}dx$$

$$V = \int dV = \int_0^L \frac{c}{4\pi\varepsilon_0}\frac{x}{(X+d)}dx$$

Let $u = x + d \rightarrow x = u - d$

$du = dx$

$$V = \int \frac{c}{4\pi\varepsilon_0} \frac{u-d}{u} du$$

$$= \frac{c}{4\pi\varepsilon_0} \int (1 - \frac{d}{u}) du$$

$$= \frac{c}{4\pi\varepsilon_0} u - d \ln u \Big|$$

$$= \frac{c}{4\pi\varepsilon_0} \left[(x+d) - d \ln(x+d)\right] \Big|_0^L$$

$$= \frac{c}{4\pi\varepsilon_0} \left\{\left[(L+d) - d \ln(L+d)\right] - \left[d - d \ln d\right]\right\}$$

$$= \frac{c}{4\pi\varepsilon_0} \left[L - d\{\ln(L+d) - \ln d\}\right]$$

$$= \frac{c}{4\pi\varepsilon_0} \left[L - d \ln\left(\frac{L+d}{d}\right)\right]$$

w/ $L = 0.12\,m$, $c = 28.9\ pC/m^2$, $d = 0.03\,m$

$$= \frac{28.9 \times 10^{-12}}{4\pi\varepsilon_0} \left[0.12 - 0.03 \ln\left(\frac{0.12 + 0.03}{0.03}\right)\right]$$

$$= \underline{\underline{1.863657 \times 10^{-2} \text{ volts}}}$$

#34

(a) $V = \frac{Q}{4\pi\varepsilon_0 L} \ln\left(\frac{d+L}{d}\right)$ (from #28 .. d is a positive number)

$$V = \frac{43.6 \times 10^{-15} c}{4\pi\varepsilon_0 (0.135m)} \ln\left(\frac{d + 0.135}{d}\right)$$

$$= \underline{\underline{2.904019886 \times 10^{-3} \ln\left(\frac{d + 0.135}{d}\right)}}$$

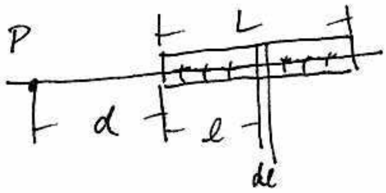
(b) $V = \frac{Q}{4\pi\varepsilon_0 L} \ln\left(\frac{x+L}{x}\right)$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\varepsilon_0 L} \frac{x}{x+L} \cdot \left(-\frac{L}{x^2}\right)$$

$$= \underline{\underline{\frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{x(x+L)}}}$$

where x is the distance between the point & the beginning of the rod (at 0,0)

Just for a quick check



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}$$

$$E = \int dE = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(-\frac{1}{x}\right)\Big|_d^{d+L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{d} - \frac{1}{d+L}\right)$$

$$= \frac{\frac{Q}{L}}{4\pi\varepsilon_0}\left(\frac{(d+L)-d}{d(d+L)}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\frac{L}{d(d+L)}$$

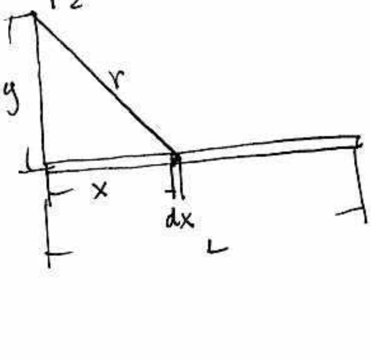
$$= \underline{\underline{\frac{Q}{4\pi\varepsilon_0}\frac{1}{d(d+L)}}}$$

(c) the direction of E_x is negative

(d) $E = \frac{43.6\times10^{-15}}{4\pi\varepsilon_0}\cdot\frac{1}{0.062(0.062+0.135)} = \underline{\underline{3.2097812\times10^{-2}\ N/C}}$

(e) symmetry ? there's no symmetry in y-direction. $dE_y = 0$ from any dq at P_1. $\therefore E_y = 0$.

#36 (since we are still using the same diagram)



(a) $dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$ $\qquad dq = \lambda dx = cx\,dx$

$$= \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{(y^2+x^2)^{1/2}}$$

$$V = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{(y^2+x^2)^{1/2}}$$

Let $u = y^2+x^2$ $\qquad x=0 \Rightarrow u=y^2$

$du = 2x\,dx$ $\qquad x=L \Rightarrow u = y^2+L^2$

$$= \frac{c}{4\pi\varepsilon_0}\int\frac{\frac{1}{2}du}{u^{1/2}}$$

$$= \frac{c}{4\pi\varepsilon_0}u^{1/2}\Big|_{y^2}^{y^2+L^2}$$

$$= \frac{c}{4\pi\varepsilon_0}\left[(y^2+L^2)^{1/2} - y\right]$$

w/ the given values

$$= \frac{49.9\ \mathrm{pC/m}}{4\pi\varepsilon_0}\left[\left((0.0356)^2 + (0.1)^2\right)^{1/2} - 0.0356\right] = \underline{\underline{3.1654176 \times 10^{-2}\ V}}$$

(b) $$E_y = -\frac{\partial V}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\left(\frac{1}{2}(y^2+L^2)^{-\frac{1}{2}} \cdot 2y - 1\right)$$

$$= \underline{\underline{\frac{c}{4\pi\varepsilon_0}\left(1 - \frac{y}{(y^2+L^2)^{1/2}}\right)}}$$

w/ the given values

$$= \frac{49.9\ \mathrm{pC/m}}{4\pi\varepsilon_0}\left(1 - \frac{0.0356}{\left((0.0356)^2+(0.1)^2\right)^{1/2}}\right) = \underline{\underline{0.29820841\ \mathrm{N/C}}}$$

(c) the V fcn we solved is a fcn of "y" although V could be a fcn of x. However, the information (to be able to describe V as a fcn of x) is not given. Hence we can not solve E_x by $-\frac{\partial V}{\partial x}$

#35 Since $E_x = -\frac{\partial V}{\partial x}$ & $E_y = -\frac{\partial V}{\partial y}$, E is a negative slope of V fcn. Fig 24-45. Both graphs show straight lines indicating E_x & E_y are constant ($\equiv$ slopes are constant)

$$E_x = -\frac{\partial V}{\partial x} = -\frac{-500\ V}{0.2\ m} = 2500\ \mathrm{N/C}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{300\ V}{0.3\ m} = -1000\ \mathrm{N/C}$$

$$\therefore\ \vec{E} = E_x\,\hat{i} + E_y\,\hat{j}$$

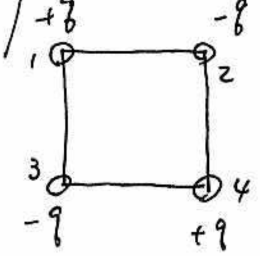
and $\qquad e^- = -1.6 \times 10^{-19}\ C$

$$\vec{F} = qE_x\,\hat{i} + qE_y\,\hat{j}$$

$$= (-1.6\times10^{-19}\ C)(2500\ \mathrm{N/C})\,\hat{i} + (-1.6\times10^{-19}\ C)(-1000\ \mathrm{N/C})\,\hat{j}$$

$$= \underline{\underline{-4\times10^{-16}\ N\,\hat{i} + 1.6\times10^{-16}\ N\,\hat{j}}} \quad (4.31\times10^{-16}\ N\ @\ -21.8^\circ)$$

#3



$$W = \overset{\curvearrowleft V \text{ created by other charges}}{V} \cdot q$$

First, imagine there was no charge.

To bring the 1st charge,

$$W_1 = \cancel{V}^{0} q_1 \quad \Rightarrow \quad \underline{W_1 = 0}$$

2nd charge

$$W_2 = V_1 q_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a}(-q) = \underline{-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}}$$

3rd charge

$$W_3 = V_1 q_3 + V_2 q_3 = \underline{\frac{1}{4\pi\varepsilon_0}\frac{q_1}{a}(-q) + \frac{1}{4\pi\varepsilon_0}\frac{-q}{\sqrt{2}a}(-q)}$$

4th charge

$$W_4 = V_1 q_4 + V_2 q_4 + V_3 q_4$$

$$= \underline{\frac{1}{4\pi\varepsilon_0}\frac{q_1}{\sqrt{2}a}(+q) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{a}\right)(+q) + \frac{1}{4\pi\varepsilon_0}\frac{-q}{a}(+q)}$$

$$W_{Total} = \Sigma W_i$$

$$= 0 + \left[-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\right] + \left[-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a}\right] + \left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\times 2\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(-4 + \frac{2}{\sqrt{2}}\right)$$

$$= \underline{\underline{\frac{1}{4\pi\varepsilon}\frac{q^2}{a}\left(-4 + \sqrt{2}\right)}}$$

$$= \frac{1}{4\pi\varepsilon}\frac{(2.30\times 10^{-12})^2}{(0.64\,m)}\left(-4 + \sqrt{2}\right) = \underline{\underline{-1.921831098\times 10^{-13}\,J}}$$

#104

(a)

$$V = \frac{Ze}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)$$

$$E = -\frac{\partial V}{\partial r} = -\frac{Ze}{4\pi\varepsilon_0}\left(-\frac{1}{r^2} + \frac{2r}{2R^3}\right)$$

$$= \underline{\underline{\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)}} \qquad \text{(This is the ans for \#83, ch.23)}$$

(b) where is $V = 0$?

$$V = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right) = 0$$

$$= \frac{Ze}{4\pi\varepsilon}\left(\frac{2R^3 - 3R^2 r + r^3}{2rR^3}\right)$$

$$\therefore \quad 2R^3 - 3R^2 r + r^3 = 0.$$

$$2R^3 - 2R^2 r - R^2 r + r^3 = 0$$

$$2R^2(R-r) - r(R^2 - r^2) = 2R^2(R-r) - r(R+r)(R-r)$$

$$= (R-r)(2R^2 - r(R+r)) = 0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$r = R \qquad 2R^2 - rR - r^2 \quad \text{or} \quad r^2 + Rr - 2R^2 = 0$$

$$r = \frac{-R \pm \sqrt{R^2 - 4(1)(-2R^2)}}{2}$$

$$r = \frac{-R \pm \sqrt{R^2 + 8R^2}}{2}$$

impossible

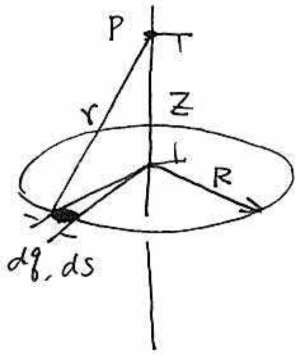
$$r = \frac{-R \pm 3R}{2} = \cancel{-2R} \text{ or } R$$

Either case $r = R$

So, $V = 0$ was set at $r = R$ instead of $r = \infty$.
(You can set $V = 0$ at anywhere you want. What's important is ΔV between two points. Not absolute V level.)

\# 114 $\quad V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} \qquad (24-32)$

(a)



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$dq = \lambda\, ds$$

$$\lambda = \frac{Q}{2\pi R}$$

$$ds = R\,d\theta$$

$$r = (R^2 + Z^2)^{1/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\,d\theta}{(R^2+Z^2)^{1/2}}$$

$$\therefore\ V = \int dV = \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0} \frac{\lambda R\, d\theta}{(R^2+z^2)^{1/2}}$$

$$= \frac{\lambda R}{4\pi\varepsilon_0} \frac{2\pi}{(R^2+z^2)^{1/2}}$$

$$= \frac{\frac{Q}{2\pi R}\cdot R}{4\pi\varepsilon_0} \frac{2\pi}{(R^2+z^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{(R^2+z^2)^{1/2}}$$

(b)

$$E = -\frac{\partial V}{\partial z} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{2} \frac{2z}{(R^2+z^2)^{3/2}}$$

$$= \frac{Q}{4\pi\varepsilon_0} \frac{z}{(R^2+z^2)^{3/2}}$$ ✓ (check w/ the eqn 22-16, P588)