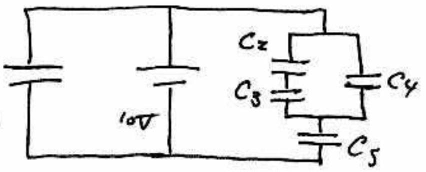
ch 25 # 12, 23, 42, 43, 44

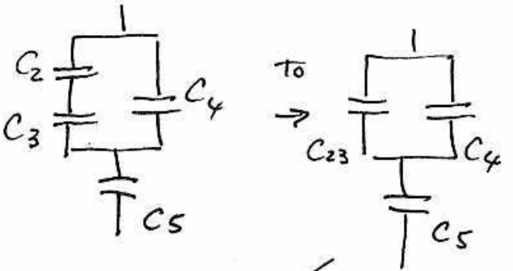
12



(a) $Q_1 = CV = 10\,\mu F \times 10\,V$

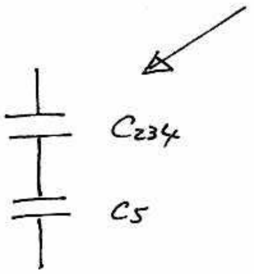
$= \underline{\underline{1 \times 10^{4}\,C}}$

(b) Before we can calculate C_2, we need to calculate C_{eq}.



$$\underset{\text{(series)}}{C_{23}} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} = \frac{C_2 C_3}{C_2 + C_3} = \frac{10\,\mu F \cdot 10\,\mu F}{10\,\mu F + 10\,\mu F}$$

$$= 5\,\mu F$$



$$\underset{\text{(parallel)}}{C_{234}} = C_{23} + C_4 = 5\,\mu F + 10\,\mu F = 15\,\mu F$$

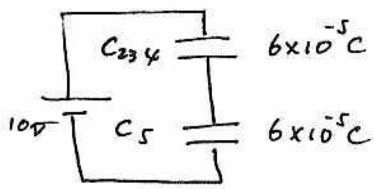
$$\underset{\text{(series)}}{C_{2345}} = \frac{1}{\frac{1}{C_{234}} + \frac{1}{C_5}} = \frac{1}{\frac{1}{15\,\mu F} + \frac{1}{10\,\mu F}} = \frac{15\,\mu F \cdot 10\,\mu F}{15\,\mu F + 10\,\mu F}$$

$$= \underline{\underline{6\,\mu F}}$$

Hence the Q in C_{eq} is

$Q = C_{eq}\,V = 6\,\mu F \times 10\,V = 6 \times 10^{-5}\,C$

Now, going back to individual caps.

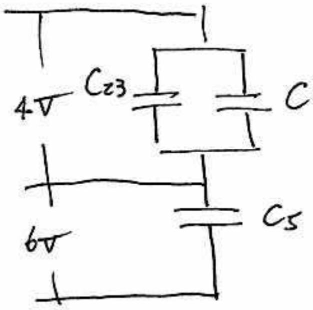


For series, $Q_{234} = Q_5 = 6 \times 10^{-5}\,C$

$\therefore$ V across C_{234} is

$$V_{(234)} = \frac{Q_{(234)}}{C_{(234)}} = \frac{6 \times 10^{-5}\,C}{15 \times 10^{-6}\,F} = 4\,V$$

(The total voltage drop across C_{234} & C_5 is $10\,V$ (= the source voltage) Hence V_5 should be $10\,V - 4\,V = 6\,V$. check this w/ $V = \frac{Q}{C} = \frac{Q_5}{C_5} = \frac{6 \times 10^{-5}\,C}{10 \times 10^{-6}\,F} = 6\,V$ ✓)



$$Q_{23} = C_{23} V_{23} = 5\times10^{-6}\,F \cdot 4V = \underline{\underline{2\times10^{-5}\,C}}$$

$$(V_{23} = V_4 = 4V)$$

Since $Q_2 = Q_3$ (series) $= Q_{23}$

$$\underline{\underline{Q_2 = 2\times10^{-5}\,C}}$$

<u>check</u>

$$V_2 + V_3 = V_4 = 4V$$

$$\frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$= \frac{2\times10^{-5}\,C}{10\,\mu F} + \frac{2\times10^{-5}\,C}{10\,\mu F} = 2V + 2V = 4V \checkmark$$

Also,

$$Q_{23} + Q_4 = 6\times10^{-5}\,C$$

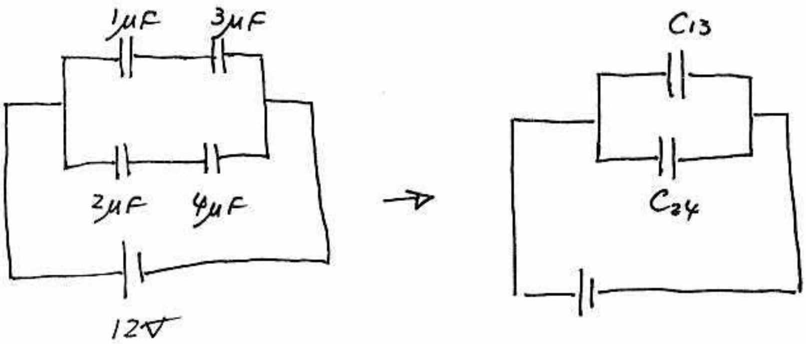
$$C_{23}V_{23} + C_4 V_4$$

$$= 2\times10^{-5}\,C + 10\,\mu F \cdot 4V$$

$$= 2\times10^{-5}\,C + 4\times10^{-5}\,C = 6\times10^{-5}\,C \checkmark$$

\# 23

(a)



$$C_{13} = \frac{1}{\frac{1}{1}+\frac{1}{3}} = \frac{3}{4}\,\mu F$$

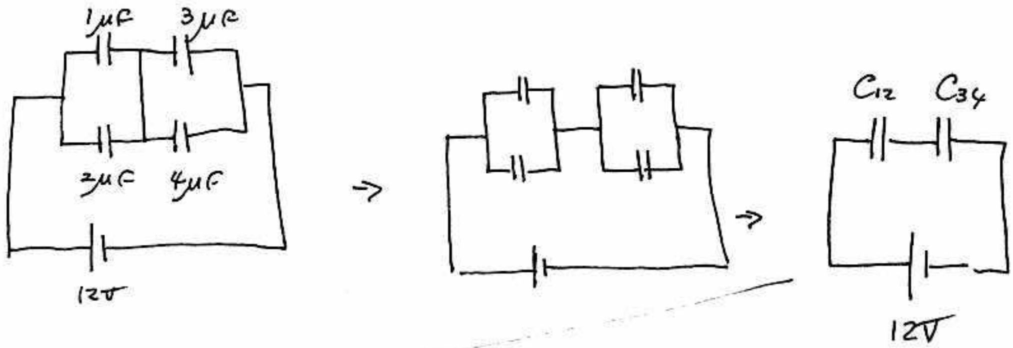
$$C_{24} = \frac{1}{\frac{1}{2}+\frac{1}{4}} = \frac{4}{3}\,\mu F$$

(d)

$$Q_{13} = C_{13}V_{13} = \frac{3}{4}\,\mu F \cdot 12V = \underline{\underline{9\times10^{-6}\,C \ (= Q_1 = Q_3)}}$$

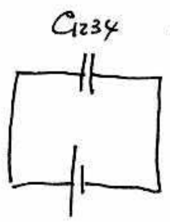
$$Q_{24} = C_{24}V_{24} = \frac{4}{3}\,\mu F \cdot 12V = \underline{\underline{16\times10^{-6}\,C \ (= Q_2 = Q_4)}}$$

(e)
~
(h)



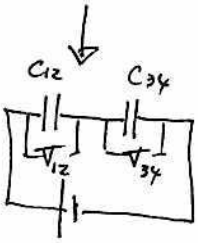
$C_{12} = C_1 + C_2$
$= 1\mu F + 2\mu F$
$= 3\mu F$

$C_{34} = C_3 + C_4$
$= 3\mu F + 4\mu F$
$= 7\mu F$



$$C_{1234} = \frac{1}{\frac{1}{3\mu F} + \frac{1}{7\mu F}} = \frac{21}{10}\,\mu F = \underline{2.1\,\mu F}$$

$$Q = CV = 2.1\,\mu F \cdot 12V = 25.2\,\mu C \quad (= Q_{12} = Q_{34})$$



$$V_{12} = \frac{Q_{12}}{C_{12}} = \frac{25.2\,\mu C}{3\,\mu F} = 8.4\,V$$

$$V_{34} = \frac{Q_{34}}{C_{34}} = \frac{25.2\,\mu C}{7\,\mu F} = 3.6\,V$$

$8.4V + 3.6V = 12V$
(equal to the source voltage)

$$Q_1 = C_1 V_1 = C_1 V_{12} = 1\,\mu F \cdot 8.4V = \underline{\underline{8.4\,\mu C}}$$

$$Q_2 = C_2 V_2 = C_2 V_{12} = 2\,\mu F \cdot 8.4V = \underline{\underline{16.8\,\mu C}}$$

$8.4\,\mu C + 16.8\,\mu C = 25.2\,\mu C$ ✓

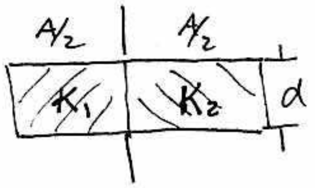
$$Q_3 = C_3 V_3 = C_3 V_{34} = 3\,\mu F \cdot 3.6\,V = \underline{\underline{10.8\,\mu C}}$$

$$Q_4 = C_4 V_4 = C_4 V_{34} = 4\,\mu F \cdot 3.6\,V = \underline{\underline{14.4\,\mu C}}$$

$10.8\,\mu C + 14.4\,\mu C = 25.2\,\mu C$ ✓

\#42

Capacitance of a parallel plate



step 1 (E)

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\,A = \frac{\sigma A}{\varepsilon_0}$$

$$\therefore E = \frac{\sigma}{\varepsilon_0}$$

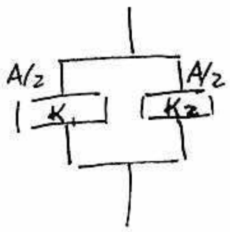
step 2 (V)

$$V = \int E \cdot dr = \int_0^d \frac{\sigma}{\varepsilon_0}\,dr = \frac{\sigma}{\varepsilon_0} d$$

step 3 ($Q = CV$, $\therefore C = \frac{Q}{V}$)

$$C_0 = \frac{Q}{\frac{\sigma}{\varepsilon}d} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}d} = \frac{A\varepsilon_0}{d}$$

Applying this to the question:



$$C_1 = K_1\frac{(\frac{1}{2}A)\,\varepsilon_0}{d} = K_1\frac{A\varepsilon_0}{2d}$$

$$C_2 = K_2\frac{(\frac{1}{2}A)\,\varepsilon_0}{d} = K_2\frac{A\varepsilon_0}{2d}$$

You can think as two parallel plate caps put them in parallel connection.

$$\therefore C_{eq} = C_1 + C_2 = K_1\frac{A\varepsilon_0}{2d} + K_2\frac{A\varepsilon_0}{2d}$$

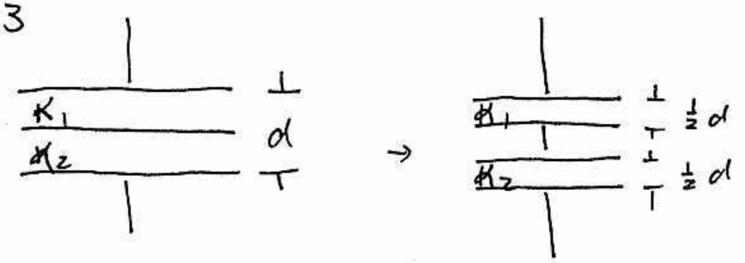
$$= \frac{A\varepsilon_0}{2d}(K_1 + K_2)$$

$$= \frac{5.56\,cm^2 \cdot \varepsilon_0}{2 \cdot 5.56\,mm}(7 + 12)$$ (Convert them into SI !!)

$$= \frac{5.56\,cm^2 \cdot \frac{1\,m^2}{(10^2)^2\,cm^2} \cdot \varepsilon_0}{2 \cdot 5.56\,mm \cdot \frac{1\,m}{10^3\,mm}}(7+1)$$

$$= \underline{\underline{8.4075 \times 10^{-12}\,F}}$$

#43



Two parallel plate caps connected in Series

step 1 $\quad E = \dfrac{\sigma}{\varepsilon_0}$ (see #42)

step 2 $\quad V = \displaystyle\int_0^{\frac{1}{2}d} E \cdot dr = \frac{\sigma}{\varepsilon_0}\left(\tfrac{1}{2}d\right)$

step 3 $\quad C = \dfrac{Q}{V} = \dfrac{Q}{\frac{\sigma}{\varepsilon_0}\frac{1}{2}d} = \dfrac{\sigma A}{\frac{\sigma d}{2\varepsilon_0}} = \dfrac{2A\varepsilon_0}{d}$

$\therefore \; C_1 = \dfrac{2K_1 A \varepsilon_0}{d} \quad \& \quad C_2 = \dfrac{2K_2 A \varepsilon_0}{d}$

$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{2K_1 A\varepsilon_0}{d}\cdot\frac{2K_2 A\varepsilon_0}{d}}{\frac{2K_1 A\varepsilon_0}{d} + \frac{2K_2 A\varepsilon_0}{d}}$$

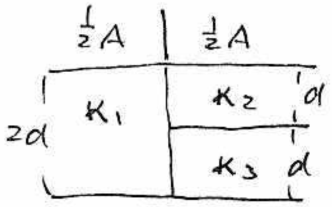
$$= \frac{2K_1 A\varepsilon_0 \; 2K_2 A\varepsilon_0}{d\,(2K_1 A\varepsilon_0 + 2K_2 A\varepsilon_0)}$$

$$= \frac{4A^2\varepsilon_0^2 K_1 K_2}{2A\varepsilon_0\, d\,(K_1+K_2)}$$

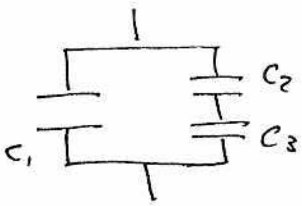
$$= \frac{2A\varepsilon_0}{d}\;\frac{K_1 K_2}{K_1+K_2} = \frac{2\cdot 7.89\ cm^2 \cdot \varepsilon_0}{4.62\ mm}\cdot\frac{11\cdot 12}{11+12}$$

$$= \frac{2\cdot 7.89\times10^{-4}\ m^2\cdot\varepsilon_0}{4.62\times10^{-3}\ m}\cdot\frac{11\cdot12}{11+12} = \underline{\underline{1.734819876\times10^{-11}\ F}}$$

#44 $\quad$ this is a combination of #42 & 43



$\Rightarrow$



$$C_1 = \frac{K_1 \cdot \frac{1}{2} A \varepsilon_0}{2d}, \quad C_2 = \frac{K_2 \cdot \frac{1}{2} A \varepsilon_0}{d}, \quad C_3 = \frac{K_3 \cdot \frac{1}{2} A \varepsilon_0}{d}$$

$$= \frac{K_1 A \varepsilon_0}{4d}, \quad = \frac{K_2 A \varepsilon_0}{2d}, \quad = \frac{K_3 A \varepsilon_0}{2d}$$

$$C_{23} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} = \frac{C_2 C_3}{C_2 + C_3} = \frac{A K_2 K_3 \varepsilon_0}{2d (K_2 + K_3)}$$

(Series)

$$C_{123} = C_1 + C_{23}$$

(parallel)

$$= \frac{K_1 A \varepsilon_0}{4d} + \frac{A K_2 K_3 \varepsilon}{2d (K_2 + K_3)}$$

$$= \frac{A \varepsilon_0 (K_1 (K_2 + K_3) + 2 K_2 K_3)}{4d (K_2 + K_3)}$$

$$= \frac{A \varepsilon_0 (K_1 K_2 + 2 K_2 K_3 + K_3 K_1)}{4d (K_2 + K_3)} = \frac{10.5 cm^2 \cdot \varepsilon_0 (21 \cdot 42 + 2 \cdot 42 \cdot 58 + 58 \cdot 21)}{4 (\frac{7.12 mm}{2})(42 + 58)}$$

$$= \frac{10.5 \times 10^{-4} m^2 \cdot \varepsilon_0 (21 \cdot 42 + 2 \cdot 42 \cdot 58 + 58 \cdot 21)}{2 (7.12 \times 10^{-3})(42 + 58)} = \underline{\underline{4.5496706 46 \times 10^{-11} F}}$$