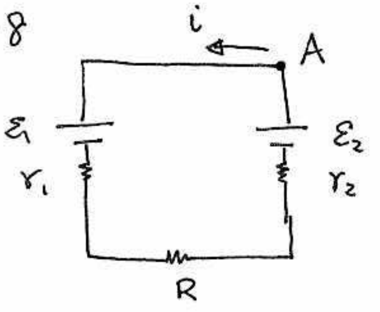
ch 27 #8, 18, 27, 41, 42, 52, 55, 67, 68, 87

#8



$\mathcal{E}_1 = 2\,V$

$\mathcal{E}_2 = 3\,V$

$r_1 = r_2 = 3\,\Omega$

$i = 1\times10^{-3}\,A$

Because $\mathcal{E}_2 > \mathcal{E}_1$, the direction of i is counter-clockwise.

(a) Loop (starting at A)

$$-\mathcal{E}_1 - ir_1 - iR - ir_2 + \mathcal{E}_2 = 0$$

$$-\mathcal{E}_1 + \mathcal{E}_2 = i(r_1 + R + r_2)$$

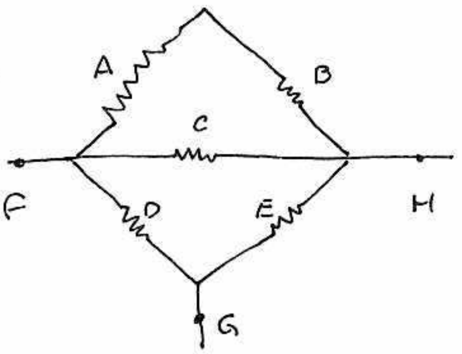
$$-2 + 3 = i(3 + R + 3)$$

$$1 = i(6 + R)$$

$$\therefore R = \frac{1}{i} - 6 = \frac{1}{1\times10^{-3}} - 6 = \underline{994\,\Omega}$$

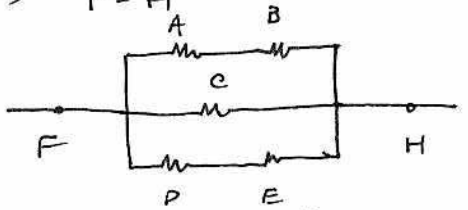
(b) $P = IV = I^2R = (1\times10^{-3})^2 \cdot 994\,\Omega = \underline{9.94\times10^{-4}\,W}$

#18

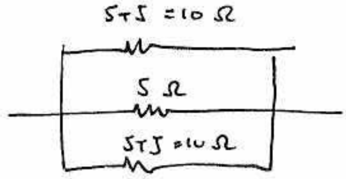


all R's are 5 Ω

(a) F - H

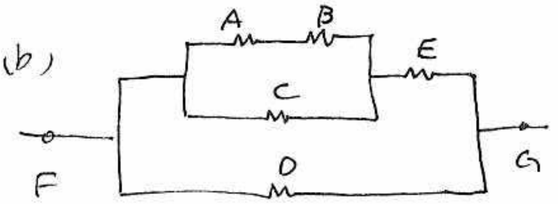


→

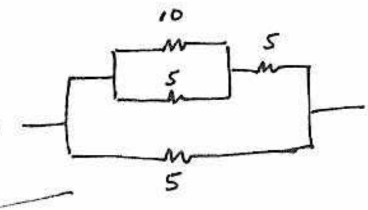


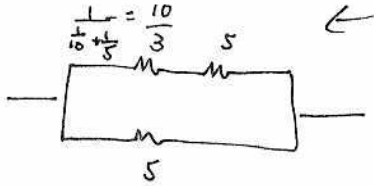
→ $R_{eq} = \dfrac{1}{\frac{1}{10}+\frac{1}{5}+\frac{1}{10}} = \dfrac{10}{4} = \underline{2.5\,\Omega}$

(b)

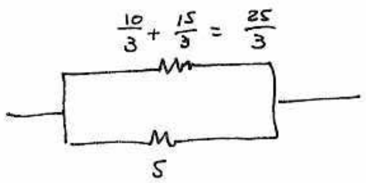


→



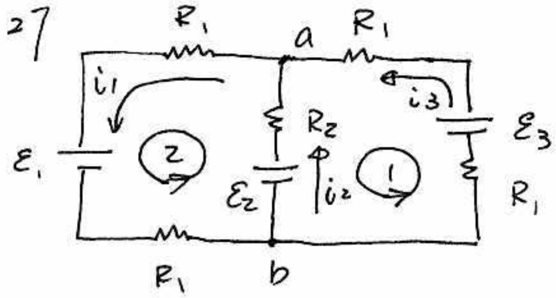


→



→ $R_{eq} = \dfrac{1}{\frac{3}{25}+\frac{1}{5}} = \underline{\dfrac{25}{8}\,\Omega}$

\# 27



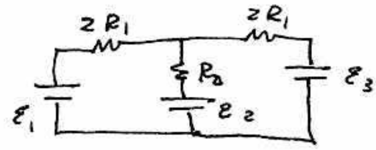
$\mathcal{E}_1 = 2\,V$

$\mathcal{E}_2 = \mathcal{E}_3 = 4\,V$

$R_1 = 1\,\Omega$

$R_2 = 2\,\Omega$

You can solve this by reducing first to



But I will do the harder way to show it still works.

Junction Rule (at a)

$$\underline{i_2 + i_3 = i_1} \quad \text{———} \quad (1)$$

Loop 1 (at a)

$$i_2R_2 - \mathcal{E}_2 - i_3R_1 + \mathcal{E}_3 - i_3R_1 = 0$$

$$2i_2 - 4 - i_3 + 4 - i_3 = 0$$

$$\therefore\ 2i_2 - 2i_3 = 0$$

$$\therefore\ \underline{i_2 = i_3} \quad \text{———} \quad (2)$$

Loop 2 (at a)

$$-i_1R_1 - \mathcal{E}_1 - i_1R_1 + \mathcal{E}_2 - i_2R_2 = 0$$

$$i_1 - 2 - i_1 + 4 - 2i_2 = 0$$

$$-2i_1 + 2 - 2i_2 = 0$$

$$\therefore\ -i_1 + 1 - i_2 = 0$$

$$\underline{i_2 = 1 - i_1} \quad \text{———} \quad (3)$$

$(1) \leftarrow (2) \leftarrow (3)$

$$(1 - i_1) + (1 - i_1) = i_1$$

$$2 - 2i_1 = i_1$$

$$3i_1 = 2$$

(a)&(b) $\quad \underline{\underline{\therefore i_1 = \frac{2}{3}\text{ amp} \text{ — } (1)' \downarrow}}$

(c)&(d) $\quad (3) \leftarrow (1)'$

$$i_2 = 1 - \frac{2}{3}\text{ ap}$$

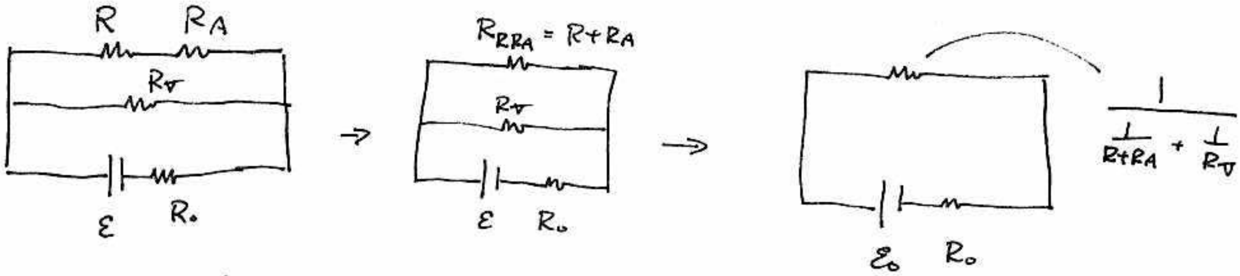
$$\underline{\underline{= \frac{1}{3}\text{ ap} \text{ — } (3)' \downarrow}}$$

(e)&(f) $\quad (2) \leftarrow (3)'$

$$\underline{\underline{i_3 = \frac{1}{3}\text{ ap} \uparrow}}$$

(g) $\quad V_a - V_b = i_2R_2 - \mathcal{E}_2 = \frac{1}{3}\cdot 2 - 4 = \underline{-\frac{10}{3}\,V}$

#41



(a) $R_T = \dfrac{R_V(R+R_A)}{R+R_A+R_V} + R_0 = \dfrac{300(85+3)}{300+85+3} + 100 = 168.0412371\ \Omega$

$i_T = \dfrac{V_T}{R_T} = \dfrac{12V}{168.041..} = \underline{\underline{7.141104295 \times 10^{-2}\ \text{amp}}}$

$V_{\left(\frac{1}{\frac{1}{R+R_A} + \frac{1}{R_V}}\right)} = i_T\ R_{\frac{1}{\frac{1}{R+R_A} + \frac{1}{R_V}}}$

$= 7.14 \ldots \dfrac{R_V(R+R_A)}{R+R_A+R_V} = \underline{4.855889570 6\ V}$

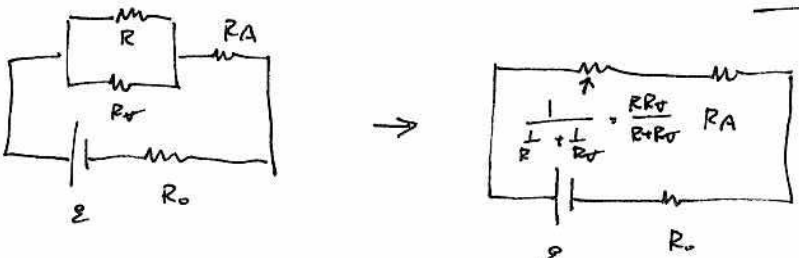
$\therefore\ i_{(\text{through } \textcircled{A})} = \dfrac{V}{R} = \dfrac{4.855..}{85+3} = \underline{\underline{5.521472393 \times 10^{-2}\ \text{ap}}}$

(b) $V = V_{\left(\frac{1}{\frac{1}{R+R_A} + \frac{1}{R_V}}\right)} = \underline{\underline{4.8558895706\ V}}$

(c) $R' = \dfrac{V'}{i} = \dfrac{4.85..}{5.52..} = \underline{\underline{88\ \Omega}}$

(d) If $R_A \downarrow$, $R_T \downarrow$, $i_T \uparrow$ $V \downarrow$, $\therefore$ $\underline{\underline{R' = \frac{V}{i} \downarrow}}$

#42



(a) $R_T = \dfrac{RR_V}{R+R_V} + R_A + R_0 = \dfrac{85 \cdot 300}{85+300} + 3 + 100 = 1.69 \times 10^2\ \Omega$

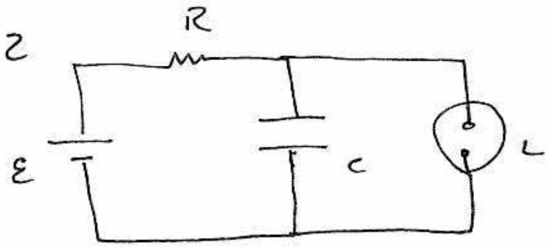
$i = \dfrac{V}{R_T} = \dfrac{12}{1.69 \times 10^2} = \underline{\underline{7.09 \times 10^{-2}\ \text{amp}}}$

(b) $V = i R_{eq} = 7.09 \times 10^{-2}\ \text{ap} \cdot \dfrac{RR_V}{R+R_V} = 4.69649 2978\ V$

(c) $R' = \dfrac{V}{i} = \dfrac{4.696\cdots}{7.09 \times 10^{-2}} = 66.23376623\ \Omega$ (way off from real R because R_V is too small)

(d) $R_V \uparrow$, $R_T \uparrow$ $i \downarrow$, $\underline{\underline{R' = \left(\frac{V}{i}\right) \uparrow}}$

#52



$C = 0.15\,\mu F$

$\mathcal{E} = 95\,V$

$V_L = 72\,V$

2 flashes / sec $\Rightarrow t = 0.5\,sec$

$$V_C = \mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$ [You should be able to derive this]

$$\frac{V_C}{\mathcal{E}} = \left(1 - e^{-\frac{t}{RC}}\right)$$

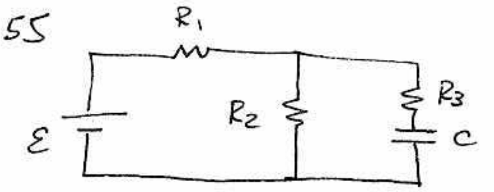
$$e^{-\frac{t}{RC}} = 1 - \frac{V_C}{\mathcal{E}}$$

$$-\frac{t}{RC} = \ln\left(1 - \frac{V_C}{\mathcal{E}}\right)$$

$$R = \frac{-t}{C\ln\left(1 - \frac{V_C}{\mathcal{E}}\right)}$$ (at $t=0$, $V_C = 0$ & $t = 0.5\,sec$, $V_C = 72\,V$)

$$= \frac{-0.5}{0.15\times10^{-6}\ln\left(1 - \frac{72}{95}\right)} = \underline{\underline{2350094.506\,\Omega}} \quad (2.35\times10^{6}\,\Omega)$$

#55

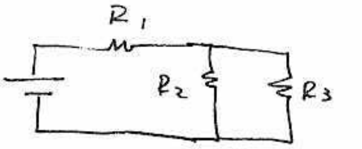


$\mathcal{E} = 1200\,V$

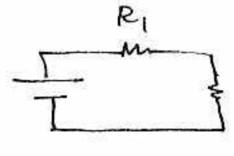
$C = 6.5\times10^{-6}\,F$

$R_1 = R_2 = R_3 = 0.73\times10^{6}\,\Omega$

(a), (b), & (c) when $t=0$, C is considered "short circuited".

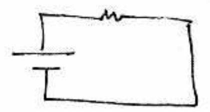


$\Rightarrow$



$\frac{1}{\frac{1}{R}+\frac{1}{R}} = \frac{R}{2}$ $\Rightarrow$

$R + \frac{R}{2} = \frac{3}{2}R$



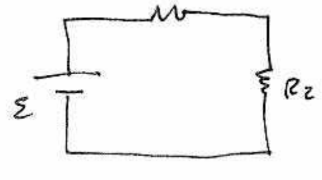
$$\therefore\ i = \frac{V}{R_{eq}} = \frac{1200}{\frac{3}{2}R} = \underline{\underline{1.095890411\times10^{-3}\,amp}} \quad (= i_1)$$

Because $R_2 = R_3$, the current will split half & half

$$i_2 = i_3 = \underline{\underline{5.479452055\times10^{-4}\,amp}} \quad (= i_2 = i_3)$$

(d), (e), & (f) when $t = \infty$, $\underline{\underline{i_3 = 0}}$ because C is fully charged.

So the circuit is considered as:

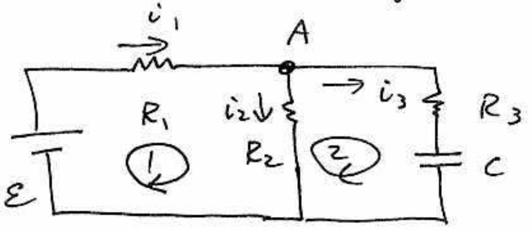


$R_{eq} = R_1 + R_2$

$$i = \frac{V}{R_{eq}} = \underline{\underline{8.219178082\times10^{-4}\,amp}} \quad (i_1 = i_2)$$

(g), (h), & (i)

You can calculate $V_2(t=0)$ by $i(t=0)R_2$ $(= 400V)$ and $V_2(t=\infty)$ by $i(t=\infty)R_2$ $(= 600V)$. But I want to show another way so that we can calculate i_2 as a fcn of time (which you should be able to do)



Junction Rule (at A)

$$i_1 = i_2 + i_3 \quad \text{———} \; ①$$

$$i_3 = \frac{dq}{dt}$$ (the current that goes a capacitor is $\frac{dq}{dt}$)

Loop I (at A)

$$-i_2R_2 + \mathcal{E} - i_1R_1 = 0 \quad \text{———} \; ②$$

Loop 2 (at A)

$$-i_3R - \frac{q}{C} + i_2R = 0 \quad \text{———} \; ③$$

② ← ①

$$-i_2R + \mathcal{E} - (i_2 + i_3)R = 0$$

$$-i_2R + \mathcal{E} - i_2R - i_3R = 0$$

$$-2i_2R - i_3R + \mathcal{E} = 0$$

$$2i_2R = \mathcal{E} - i_3R$$

$$\therefore \; i_2 = \frac{\mathcal{E} - i_3R}{2R} \quad \text{———} \; ②'$$

③ ← ②'

$$-i_3R - \frac{q}{C} + \left(\frac{\mathcal{E} - i_3R}{2R}\right)R = 0$$

$$-i_3R - \frac{q}{C} + \frac{\mathcal{E}}{2} - \frac{i_3R}{2} = 0$$

$$-\frac{3}{2}i_3R - \frac{q}{C} + \frac{\mathcal{E}}{2} = 0$$ [We want solve for i_3 since $i_3 = \frac{dq}{dt}$]

$$\therefore \; i_3 = \frac{2}{3R}\left(-\frac{q}{C} + \frac{\mathcal{E}}{2}\right) = \frac{2}{3R}\left(\frac{-2q + \mathcal{E}C}{2C}\right)$$

$$= \frac{-1}{3RC}(2q - \mathcal{E}C)$$

$$= \frac{-2}{3RC}\left(q - \frac{\mathcal{E}C}{2}\right) = \frac{dq}{dt}$$

$$\int \frac{-2}{3RC}\,dt = \int \frac{1}{q-\frac{\varepsilon C}{2}}\,dq$$

$$e^{\frac{-2t}{3RC}} = e^{\ln\left(q-\frac{\varepsilon C}{2}\right)+\gamma}$$

$$e^{-\frac{2t}{3RC}} = \left(q-\frac{\varepsilon C}{2}\right)\gamma'$$

when $t=0$, $q=0$

$$\therefore\ 1 = -\frac{\varepsilon C}{2}\gamma'$$

$$\therefore\ \gamma' = -\frac{2}{\varepsilon C}$$

So the fcn is

$$e^{\frac{-2t}{3RC}} = \left(q-\frac{\varepsilon C}{2}\right)\left(-\frac{2}{\varepsilon C}\right)$$

$$\therefore\ q = \frac{\varepsilon C}{2}\left(1-e^{-\frac{2t}{3RC}}\right)$$

So, $$i_3 = \frac{dq}{dt} = \frac{\varepsilon C}{2}\,\frac{2}{3RC}\,e^{\frac{-2t}{3RC}} = \frac{\varepsilon}{3R}e^{\frac{-2t}{3RC}} \quad \text{—— ③}'$$

$$②' \longleftarrow ③'$$

$$i_2 = \frac{\varepsilon-\left(\frac{\varepsilon}{3R}e^{\frac{-2t}{3RC}}\right)R}{2R} = \frac{\varepsilon}{2R}\left(1-\frac{1}{3}e^{-\frac{2t}{3RC}}\right)$$

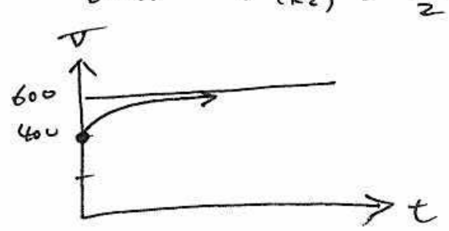
$$\therefore\ V_{(R_2)} = i_2R_2$$

$$= \frac{\varepsilon}{2}\left(1-\frac{1}{3}e^{-\frac{2t}{3RC}}\right)$$

when $t=0$ $\quad V_{(R_2)} = \frac{\varepsilon}{2}\left(1-\frac{1}{3}e^{0}\right) = \frac{1}{3}\varepsilon = 400V$

$t=\infty$ $\quad V_{(R_2)} = \frac{\varepsilon}{2}\left(1-\frac{1}{3}e^{-\infty}\right) = \frac{1}{2}\varepsilon = 600V$

they match !!



(Extra) $\tau_c = \frac{3}{2}RC$ in this case and $\infty \sim 6RC$

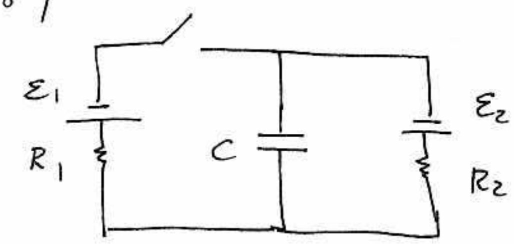
So $\infty = 6RC = 6\cdot\frac{3}{2}RC$

$= 9\,(0.73\times10^{6})(6.5\times10^{-6})$

$= 42.705\ \text{sec}$

So Physicsly, $t \geq 42.705\ \text{sec}$ is ∞ for this circuit.

#89



$C = 10\,\mu F$

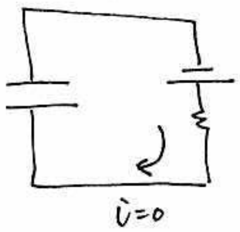
$\mathcal{E}_1 = 1\,V$

$\mathcal{E}_2 = 3\,V$

$R_1 = 0.2\,\Omega$

$R_2 = 0.4\,\Omega$

<u>First condition</u>



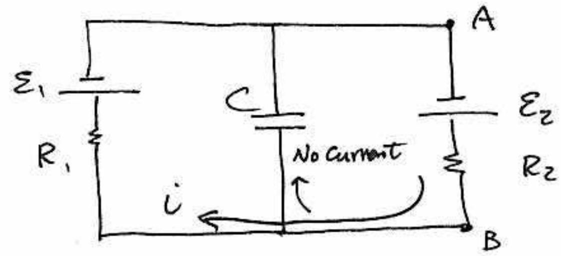
Since the cap is fully charged, $i = 0$

and $V_C = \mathcal{E}$

$\therefore \quad Q_i = CV$

$= 10\,\mu F \cdot 3V = \underline{\underline{30\,\mu C}}$

<u>Second condition</u>



<u>Loop 1 at A</u>

$\mathcal{E}_2 - iR_2 - iR_1 - \mathcal{E}_1 = 0$

$\mathcal{E}_2 - \mathcal{E}_1 - i(R_1 + R_2) = 0$

$3 - 1 - i(0.2 + 0.4) = 0$

$2 - i(0.6) = 0$

$\therefore \quad i = \frac{2}{0.6} = \frac{10}{3}$ amp

$V_C = V_{AB} = \mathcal{E}_2 - iR_2$

$= 3 - \frac{10}{3} \cdot 0.4 = \underline{\frac{5}{3}V}$

$\therefore \quad Q_f = CV$

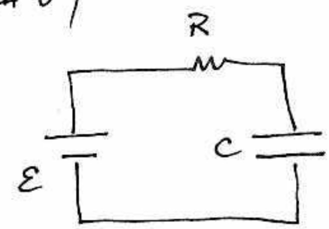
$= 10\,\mu F \cdot \frac{5}{3}V = \underline{\underline{\frac{50}{3}\mu C}}$

$\Delta Q = Q_f - Q_i = \frac{50}{3}\mu C - 30\,\mu C$

$= \frac{50}{3}\mu C - \frac{90}{3}\mu C$

$= \underline{\underline{-\frac{40}{3}\mu C}}$

#67



(a) Energy stored in the cap. when it is fully charged

$$U = \frac{1}{2} C V^2 \qquad q = CV \rightarrow C = \frac{q}{V}$$

$$= \frac{1}{2} \frac{q}{V} \cdot V^2$$

$$= \underline{\underline{\frac{1}{2} q V^2}}$$

While the charge "q" is moving to the cap., the work or energy released by the batery is

$$W = \underline{\underline{qV}}$$

Hence a half of work done by the batery is stored in the cap. (the other half? It was used up by the resistor)

(b)

$$V = V_0 \left(1 - e^{-\frac{t}{RC}}\right)$$

(I started w/ this, but you should be able to derive this w/o any trouble)

since

$$P = I^2 R \quad \& \quad W = \int P \cdot dt,$$

$$W = \int I^2 R \cdot dt.$$

We need to get $I(t)$.

$$q = CV = CV_0 \left(1 - e^{-\frac{t}{RC}}\right)$$

$$\therefore i(t) = \frac{dq}{dt} = \frac{d\left(CV_0\left(1 - e^{-\frac{t}{RC}}\right)\right)}{dt} = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

$$\therefore W = \int i^2 R \, dt$$

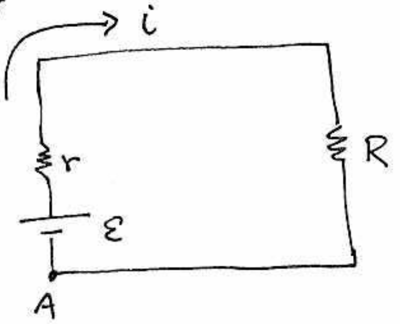
$$= \int \left(\frac{V_0}{R} e^{-\frac{t}{RC}}\right)^2 \cdot R \, dt$$

$$= \int \frac{V_0^2}{R} e^{-\frac{2t}{RC}} \cdot dt$$

$$= \frac{V_0^2}{R} \left(-\frac{RC}{2}\right) e^{-\frac{2t}{RC}} \Big|_0^\infty$$

$$= \frac{V_0^2 C}{2} = \frac{1}{2} C V_0^2 \quad \ldots \text{ the same result.}$$

#68



starting at A

$$\varepsilon - ir - iR = 0$$

$$V = i(r + R)$$

$$i = \frac{V}{r+R} \quad \text{——————} \ ①$$

$$P = iV = i^2 R \quad \text{——————} \ ②$$

$$② \leftarrow ①$$

$$P = \left(\frac{V}{r+R}\right)^2 R \quad \text{——————} \ ②'$$

$$= V^2 \cdot \frac{R}{(r+R)^2}$$

To maximize P, set $\frac{dP}{dR} = 0$ (Because V & r are const. only R is a factor to decide P)

$$\frac{dP}{dR} = \frac{d\left(V^2 \cdot \frac{R}{(r+R)^2}\right)}{dR} = V^2 \left(\frac{1}{(r+R)^2} - \frac{2R}{(r+R)^3}\right)$$

$$= V^2 \left(\frac{(r+R) - 2R}{(r+R)^3}\right)$$

$$= V^2 \left(\frac{r-R}{(r+R)^3}\right) = 0$$

$$\therefore \ r - R = 0 \quad \text{or} \quad \underline{\underline{r = R}} \quad \text{——————} \ ②''$$

$$②' \leftarrow ②''$$

$$P = \left(\frac{V}{r+r}\right)^2 \cdot r$$

$$= \left(\frac{V}{2r}\right)^2 \cdot r$$

$$= \frac{V^2}{4r^2} \cdot r$$

$$= \underline{\underline{\frac{V^2}{4r}}}$$