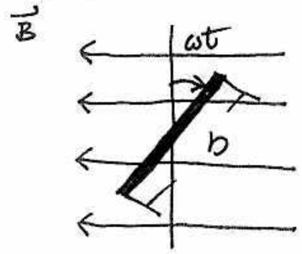
Ch. 30 #11, 24, 30, 32, 33, 39, 41, 45, 46, 53, 62, 68, 72, 89, 92

#11. seen from the edge

(a)



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d\int B \cdot dA}{dt}$$

$$= -\frac{d[N\,\overbrace{(B \cdot (ab)\cos\theta)}^{\text{mag. flux per turn}}]}{dt}$$

$$= -\frac{d\,[NB\,ab\cos(\omega t)]}{dt}$$

$$= \omega NB\,ab\sin(\omega t)$$

since $\omega = 2\pi\nu$

$$= \underbrace{2\pi\nu NB\,ab}\sin(2\pi\nu t)$$

$$= \mathcal{E}_0 \sin(2\pi\nu t)$$

(b)

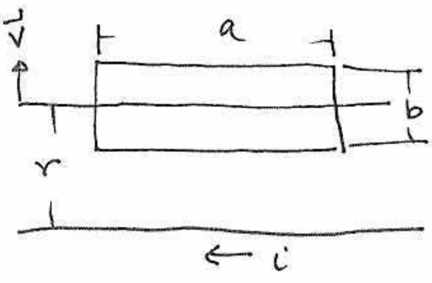
$$\mathcal{E}_0 = 2\pi\nu NB(ab) = 150\,V$$

$$60\ \text{rev/sec} = 60\ Hz = 60\ \frac{rev}{sec}\cdot\frac{2\pi\ rad}{rev} = 120\pi\ rad/sec$$

$$150\,V = \omega NB\,ab$$

$$Nab = \frac{150\,V}{\omega B} = \frac{150\,V}{120\pi\ rad/sec \cdot 0.5\,T} = 0.795774715\ m^2$$

#24



The direction of the induced i is clockwise

(a) $\Phi_B = \int B \cdot dA$

$= \int \frac{\mu_0 i}{2\pi\delta}\, a \cdot d\delta$ We did this in the lecture. If you are not sure how this is done, see your lecture note

$$= \frac{\mu_0 i a}{2\pi} \ln\delta \Big|_{r-b/2}^{r+b/2}$$

$$= \frac{\mu_0 i a}{2\pi} \ln\frac{r+\frac{b}{2}}{r-\frac{b}{2}}$$

$a = 2.2\ cm = 0.022\ m$

$b = 0.8\ cm = 0.008\ m$

$R = 0.4\ m\Omega = 4\times10^{-4}\ \Omega$

$i = 4.7\ A$

$V = 3.2\ mm/sec = 3.2\times10^{-3}\ m/sec$

$r = 1.5b$

$$= \frac{4\pi\times10^{-7}\cdot 4.7\cdot 0.022}{2\pi}\ln\left[\frac{1.5(0.008)+\frac{0.008}{2}}{1.5(0.008)-\frac{0.008}{2}}\right]$$

$$= 0.143342836\times10^{-8}\ wb$$

Make sure to use MSK (meter-second-kilogram)!!

(b) $\mathcal{E}_{ind} = -\dfrac{d\Phi_0}{dt}$

$$= -\frac{d\left[\frac{\mu_0 i a}{2\pi} \ln \frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right]}{dt}$$

$$= -\frac{\mu_0 i a}{2\pi}\left[\frac{1}{r+\frac{b}{2}}\cdot\frac{dr}{dt} - \frac{1}{r-\frac{b}{2}}\cdot\frac{dr}{dt}\right]$$

$$= -\frac{\mu_0 i a}{2\pi}\left[\frac{1}{r+\frac{b}{2}} - \frac{1}{r-\frac{b}{2}}\right]\cancel{\frac{dr}{dt}}\, v$$

$$= -\frac{\mu_0 i a v}{2\pi}\left[\frac{(r-\frac{b}{2})-(r+\frac{b}{2})}{(r+\frac{b}{2})(r-\frac{b}{2})}\right]$$

$$= -\frac{\mu_0 i a v}{2\pi}\ \frac{-b}{r^2-(\frac{b}{2})^2}$$

$$= \frac{\mu_0 i a b v}{2\pi}\ \frac{1}{(r^2-\frac{b^2}{4})}$$

$$= \frac{\mu_0 i a b v}{2\pi}\ \frac{1}{\frac{1}{4}(4r^2-b^2)}$$

$$= \frac{2\mu_0 i a b v}{\pi}\cdot\frac{1}{(4r^2-b^2)}$$

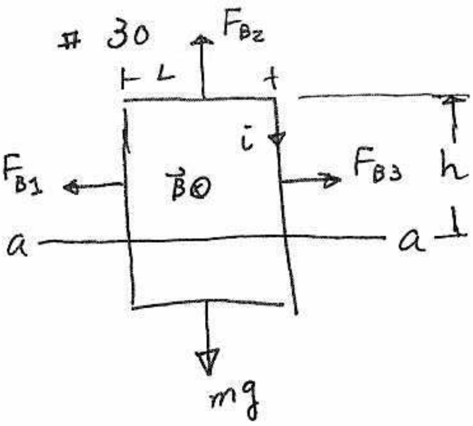
$$= \frac{2\cdot 4\pi\times10^{-7}\cdot(4.7)(0.022)(0.008)(3.2\times10^{-3})}{\pi}\ \frac{1}{(4(1.5b)^2-b^2)}$$

$$= 8\times10^{-7}(4.7)(0.022)(0.008)(3.2\times10^{-3})\cdot\frac{1}{8(0.008)^2}$$

$$= 4.136\times10^{-9}\ V$$

$$\therefore\ i = \frac{\mathcal{E}}{R} = \frac{4.136\times10^{-9}\ V}{4\times10^{-4}\ \Omega} = \underline{\underline{1.034\times10^{-5}\ \text{amp}}}$$

30



x cop

$$\vec{F}_{B1} + \vec{F}_{B3} = 0$$

y cop

$$F_{B2} - mg = ma$$

Also

$F_{B2} = i_{ind} \cdot L\,B$ & $a = 0$ for terminal speed.

$$\therefore \quad F_{B2} - mg = \cancel{ma}^{\,0}$$

$$i_{ind} L B = mg \quad \text{———} \; ①$$

Induced "i"

$$i = \frac{\varepsilon}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{1}{R}\,\frac{-d\int B \cdot dA}{dt} = \frac{-1}{R}\frac{d(BLh)}{dt}$$

$$= -\frac{1}{R}\, BL\, \frac{dh}{dt} \; (-V)$$

(h is getting shorter)

$$= \frac{BLV}{R} \quad \text{———} \; ②$$

$① \leftarrow ②$

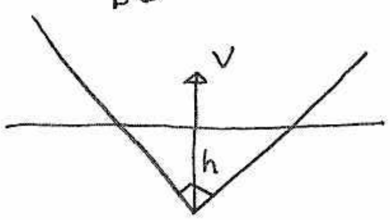
$$\frac{BLV}{R} \cdot L\,B = mg$$

$$\frac{B^2L^2V}{R} = mg$$

$$\therefore \quad V = \frac{mgR}{B^2L^2}$$

32

$B\odot$



at $t=0$, $V = 5.2$ m/sec w/ $a = 0$ m/sec²

$B = 0.35\,T$

For this triangle base is "2h" & height is "h"

(a) at $t = 3$ sec, $h = 5.2$ m/sec $\cdot$ 3 sec $= 15.6$ m

$$\Phi_B = \int B \cdot dA$$

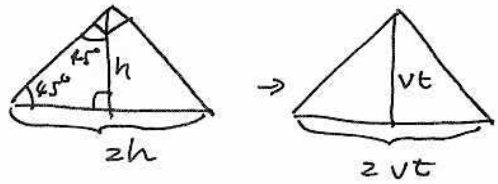
$$= B \cdot \text{Area} = B\,\tfrac{1}{2}(2h \cdot h) = Bh^2$$

$$= 0.35\,(15.6)^2 = \underline{\underline{85.176\ \text{wb}}}$$

(b) $$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d(Bh^2)}{dt} = \left(B\,2h\,\frac{dh}{dt}\right)$$

$$= 0.35 \cdot 2 \cdot (15.6) \cdot 5.2 = \underline{\underline{56.784\ V}}$$

(c) $h = vt$



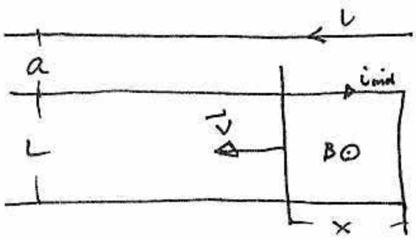
$$\text{area} = \frac{1}{2}(2vt)(vt) = (vt)^2$$

$$\therefore \Phi_B = B\cdot(vt)^2$$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(Bv^2t^2)}{dt} = Bv^2\cdot 2t = 2Bv^2\cdot t = at^b$$

$$\boxed{\therefore\ a = 2Bv^2 \quad b = 1}$$

33



"i_{ind} is clockwise"

$L = 0.1\,m$

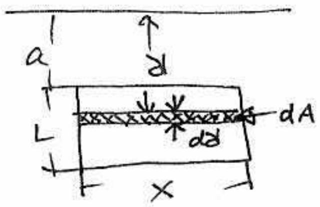
$V = 5\,m/sec$

$R = 0.4\,\Omega$

$i = 100\,A$ (I wanna see the wire ... not a cable!)

$a = 0.01\,m$

(a) $$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d\int B\cdot dA}{dt} = \frac{d\int \frac{\mu_0 i}{2\pi r}\cdot dr\cdot x}{dt}$$



$$= \frac{d\left[\frac{\mu_0 i x}{2\pi}\ln r\Big|_a^{a+L}\right]}{dt}$$

$$= \frac{d\left[\frac{\mu_0 i x}{2\pi}\ln\frac{a+L}{a}\right]}{dt}$$

$$= \frac{\mu_0 i}{2\pi}\ln\left|\frac{a+L}{a}\right|\cdot\frac{dx}{dt}^{\,v}$$

$$= \frac{\mu_0 i}{2\pi}\ln\frac{a+L}{a}\cdot v$$

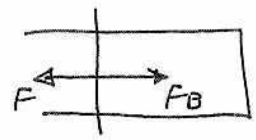
$$= \frac{4\pi\times10^{-7}\cdot 100}{2\pi}\cdot\ln\frac{0.01+0.1}{0.01}\cdot 5$$

$$= \underline{\underline{2.397895273\times10^{-4}\,V}}$$

(b) $$i = \frac{\mathcal{E}_{ind}}{R} = \frac{2.397\cdots\times10^{-4}\,V}{0.4\,\Omega} = \underline{\underline{5.994738182\times10^{-4}\,\text{amp}}}$$

(c) $$P = i^2R = \underline{\underline{1.437475435\times10^{-7}\,\text{watt}}}$$

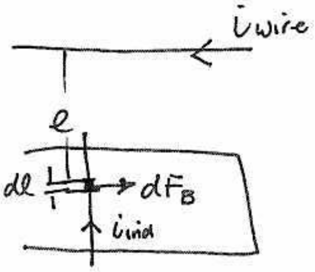
(d)



Since V is const, $a = 0$.

$$\therefore \quad F - F_B = 0$$

$$F = F_B$$



$$dF_B = i_{ind}\, dl\, B_{\text{by the wire at the loop}}$$

$$= i_{ind}\, dl\, \frac{\mu_0 i_{wire}}{2\pi l}$$

Make sure you know which "i" is which. This is a typical beginner's mistake

$$\therefore F_B = \int dF_B = \int i_{ind} \frac{\mu_0 i_{wire}}{2\pi l}\, dl$$

$$= \frac{\mu_0 i_{wire}\, i_{ind}}{2\pi} \cdot \ln l \Big|_a^{a+L}$$

$$= \frac{\mu_0 i_{wire}\, i_{ind}}{2\pi} \ln \frac{a+L}{a}$$

$$= \frac{4\pi \times 10^{-7} \cdot (100)(5.9947\ldots \times 10^{-4})}{2\pi} \ln\left(\frac{0.01+0.1}{0.01}\right)$$

$$= \underline{\underline{2.87495087 \times 10^{-8}\, N}}$$

(e) From Physics 230.

$$P = \frac{dW}{dt} = \frac{d\int F \cdot dr}{dt}$$

$$= \frac{d(Fr)}{dt} \quad (\text{for const. } F)$$

$$= F \underbrace{\frac{dr}{dt}}_{V}$$

$$= (2.8749\ldots \times 10^{-8} N)(5\ m/\text{sec})$$

$$= \underline{1.437475435 \times 10^{-7}\ \text{watt}}$$

(c) & (e) must agree because the thermal energy has to be provided by some outside source. Energy put in (from outside) = Energy spent (as thermal energy via R)

per turn

39

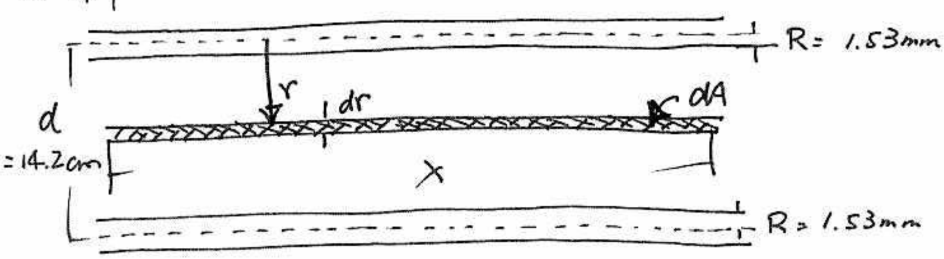
$$L\frac{di}{dt} = \frac{N\, d\int B\cdot dA}{dt}$$

$$\int L\, di = \int N\, d\Phi_B$$

$$Li = N\Phi_B$$

$$\Phi_B = \frac{Li}{N} = \frac{8\times10^{-3}\cdot 5\times10^{-3}}{400} = \frac{40\times10^{-6}}{400} = \underline{\underline{1\times10^{-7}\ \text{Wb}}}$$

41



$$\Phi_B = \int B\cdot dA$$

$$= \int_R^{d-R} \frac{\mu_0 i}{2\pi r}\cdot x\, dr$$

$$= \frac{\mu_0 i x}{2\pi}\ln r\Big|_R^{d-R}$$

from both wires

$$= \frac{\mu_0 i x}{2\pi}\ln\frac{d-R}{R} \Rightarrow \Phi_{B,\text{Total}} = 2\times\Phi_B = \frac{\mu_0 i x}{\pi}\ln\frac{d-R}{R}$$

$$L\frac{di}{dt} = \frac{d\Phi_B}{dt}$$

$$\int L\, di = \int d\Phi_B$$

$$Li = \frac{\mu_0 i x}{\pi}\ln\frac{d-R}{R}$$

$$L = \frac{\mu_0 x}{\pi}\ln\frac{d-R}{R}$$

$$\frac{L}{x} = \frac{\mu_0}{\pi}\ln\frac{d-R}{R}$$

$$= \frac{4\pi\times10^{-7}}{\pi}\ln\left(\frac{142-1.53}{1.53}\right) = \underline{\underline{1.807890483\times10^{-6}\ \text{H/m}}}$$

45

$$V_1 + V_2 = V_T$$

(a)

$$L_1 \frac{di}{dt} + L_2 \frac{di}{dt} = L_{eq} \frac{di}{dt} \quad \left(\frac{di}{dt} \text{ is the same for all} \right)$$

$$\therefore \; \underline{\underline{L_1 + L_2 = L_{eq}}}$$

(b) $\underline{\underline{L_{eq} = \Sigma_i L_i}}$

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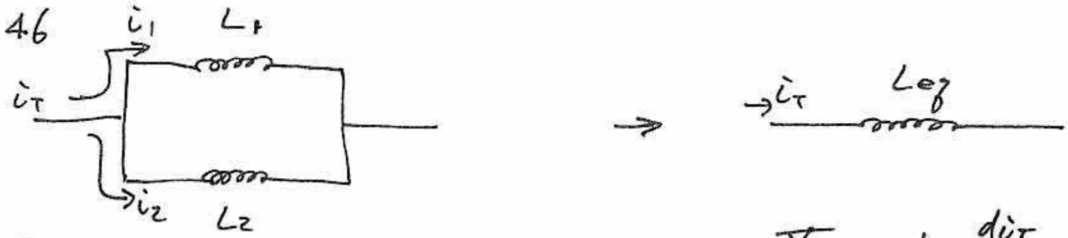


(a)

$$V_1 = L_1 \frac{di_1}{dt} \Rightarrow \frac{di_1}{dt} = \frac{V_1}{L_1} \text{ --- } ①$$

$$V_2 = L_2 \frac{di_2}{dt} \Rightarrow \frac{di_2}{dt} = \frac{V_2}{L_2} \text{ --- } ②$$

$$V_{eq} = L_{eq} \frac{di_T}{dt}$$

$$= L_{eq} \frac{d(i_1 + i_2)}{dt}$$

$$= L_{eq} \left(\frac{di_1}{dt} + \frac{di_2}{dt} \right) \text{ --- } ③$$

③ ← ① & ②

$$V_{eq} = L_{eq} \left(\frac{V_1}{L_1} + \frac{V_2}{L_2} \right)$$

$$\frac{V_{eq}}{L_{eq}} = \frac{V_1}{L_1} + \frac{V_2}{L_2}$$

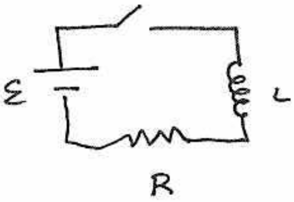
However $V_1 = V_2 = V_{eq}$

$$\therefore \; \underline{\underline{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}}$$

(b)

$$\underline{\underline{L_{eq} = \frac{1}{\Sigma \frac{1}{L_i}}}}$$

#53



$\mathcal{E} = 14\,V$

$L = 6.3 \times 10^{-6}\,H$

$R = 1.2 \times 10^{3}\,\Omega$

<u>Loop</u>

$$\mathcal{E} - L\frac{di}{dt} - iR = 0$$

$$\mathcal{E} - iR = L\frac{di}{dt}$$

$$i - \frac{\mathcal{E}}{R} = -\frac{L}{R}\frac{di}{dt}$$

$$\frac{i - \frac{\mathcal{E}}{R}}{-L/R} = \frac{di}{dt}$$

$$\int \frac{dt}{-L/R} = \int \frac{di}{i - \frac{\mathcal{E}}{R}}$$

$$e^{\frac{t}{-L/R} + \gamma} = e^{\ln\left(i - \frac{\mathcal{E}}{R}\right)}$$

$$e^{-\frac{t}{L/R}} \cdot \gamma' = i - \frac{\mathcal{E}}{R}$$

at $t = 0$, $i = 0$

$$\therefore \gamma' = -\frac{\mathcal{E}}{R}$$

$$\therefore e^{-\frac{t}{L/R}} \cdot \left(-\frac{\mathcal{E}}{R}\right) = i - \frac{\mathcal{E}}{R}$$

$$\therefore i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

(a)

$$i = 0.8\, i_{max} = 0.8\frac{\mathcal{E}}{R}$$

$$0.8\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

$$0.2 = e^{-\frac{t}{L/R}}$$

$$\ln 0.2 = -\frac{t}{L/R}$$

$$\therefore t = -\frac{L}{R}\ln 0.2$$

$$= -\frac{6.3 \times 10^{-6}}{1.2 \times 10^{3}}\ln 0.2$$

$$= 8.44954904 \times 10^{-9}\ \text{sec}$$

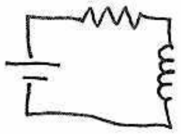
(b) at $t = 1\tau_L$

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1}\right)$$

$$= \frac{14}{1.2 \times 10^{3}}\left(1 - e^{-1}\right)$$

$$= 7.374739853 \times 10^{-3}\ \text{amp}$$

#62



$\mathcal{E} = 10\ V$
$R = 6.7\ \Omega$
$L = 5.5 H$

From #53,

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

(a) $W = q\mathcal{E}$ (q is the total charge left from the battery between 0 – 2 sec)

$$= \mathcal{E}\int dq \qquad \left(i = \frac{dq}{dt}\right)$$

$$= \mathcal{E}\int i\,dt$$

$$= \mathcal{E}\int \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right) . dt$$

$$= \frac{\mathcal{E}^2}{R}\left(t + \frac{L}{R} e^{-\frac{t}{L/R}}\right)\Big|_0^2$$

$$= \frac{10^2}{6.7}\left[\left(2 + \frac{5.5}{6.7} e^{-\frac{2}{5.5/6.7}}\right) - \left(0 + \frac{5.5}{6.7} e^0\right)\right]$$

$$= \underline{\underline{18.67037441\ J}}$$

(b) $E_B = \frac{1}{2} L i^2$

$$= \frac{1}{2}\, 5.5\left(\frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)\right)^2 \quad \text{at } t = 2 \text{ sec}$$

$$= \underline{\underline{5.10116536\ J}}$$

(c) $P = i^2 R$

$$= \left[\frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)\right]^2 R$$

$$= \frac{\mathcal{E}^2}{R}\left(1 - e^{-\frac{t}{L/R}}\right)^2$$

$$= \frac{\mathcal{E}^2}{R}\left(1 - 2e^{-\frac{t}{L/R}} + e^{-\frac{2t}{L/R}}\right)$$

$$W = \int P . dt = \int \frac{\mathcal{E}^2}{R}\left(1 - 2e^{-\frac{t}{L/R}} + e^{-\frac{2t}{L/R}}\right) dt$$

$$= \frac{\mathcal{E}^2}{R}\left(t + \frac{2L}{R} e^{-\frac{t}{L/R}} - \frac{L}{2R} e^{-\frac{t}{L/R}}\right)\Big|_0^2$$

$$= \frac{10^2}{6.7}\left[\left(2 + \frac{2(5.5)}{6.7} e^{-\frac{2}{5.5/6.7}} - \frac{5.5}{2(6.7)} e^{-\frac{2}{5.5/6.7}}\right) - \left(0 + \frac{2(5.5)}{6.7} e^0 - \frac{5.5}{2(6.7)} e^0\right)\right]$$

$$= \underline{\underline{13.50187263\ J}}$$

check:

(a) = (b) + (c)

Energy provided by the battery = Magnetic energy + thermal energy wasted by R

\# 68

$$\mathcal{E} = M \frac{di}{dt}$$

$$30\times10^{3}\,V = M \frac{6A}{2.5\times10^{-3}\,sec}$$

$$M = \frac{30\times10^{3}\,V \cdot 2.5\times10^{-3}\,sec}{6\,A}$$

$$= \underline{\underline{12.5\ H}}$$

\# 72

(a) $B_{phys} = \mu_0 i n$ (You should be able to derive this)

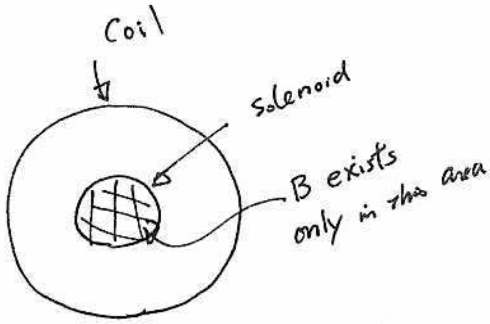
$$\Phi_B = \int B \cdot dA = \mu_0 i n \cdot \pi R^2 \quad \text{(per turn of C)}$$

$$\therefore \Phi_{B\,Total} = N \mu_0 i n \pi R^2$$

$$M \frac{di}{dt} = \frac{d\Phi_{B\,Total}}{dt}$$

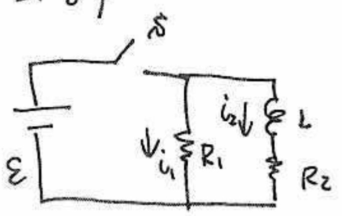
$$M i = N \mu_0 i n \pi R^2$$

$$\therefore \underline{\underline{M = N \mu_0 n \pi R^2}}$$



(b) B created by S is uniform and it exists only inside the S. So even if the shape of C changes, Φ_B will not change as long as S is inside of C.

\# 89



$\mathcal{E} = 10V$
$R_1 = 5\Omega$
$R_2 = 10\Omega$
$L = 5.0H$

(a) $i_1 = \frac{\mathcal{E}}{R} = \frac{10}{5} = \underline{\underline{2 amps}}$

(b) at $t=0$, $L\frac{di}{dt} = 10V \rightarrow \underline{i_2 = 0 amp}$

(c) $i_T = i_1 + i_2 = 2 + 0 = \underline{\underline{2 amps}}$

(d) $V_2 = i_2 R_2 = \underline{\underline{0V}}$

(e) $V_L = L\frac{di}{dt} = \underline{\underline{10V}}$ ($\mathcal{E} - L\frac{di_2}{dt} - \cancel{i_2 R_2}^{0} = 0$

(f) $\frac{di_2}{dt} = \frac{10V}{L} = \frac{10V}{5H} = \underline{\underline{2\ amp/sec}}$

(g) at $t = \infty$

$$i_1 = \frac{\varepsilon}{R_1} = \frac{10\,V}{5\,\Omega} = \underline{\underline{2\text{ amp}}}$$

h) $$i_2 = \frac{\varepsilon}{R_2} = \frac{10\,V}{10\,\Omega} = \underline{\underline{1\text{ amp}}}$$

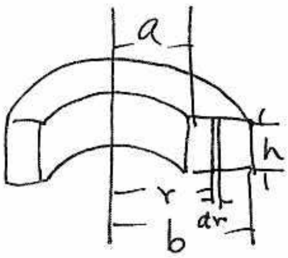
(i) $$i_T = i_1 + i_2 = 2 + 1 = \underline{\underline{3\text{ amps}}}$$

(j) $$V_2 = i_2 R_2 = \underline{\underline{10\,V}}$$

(k) $$V_L = \underline{\underline{0\,V}}$$

(l) $$\frac{di_2}{dt} = \frac{0\,V}{L} = \underline{\underline{0\text{ amp/sec}}}$$

#92



$$\oint B \cdot ds = \mu_0 i_{enc}$$

$$B \cdot 2\pi r = N_1 \mu_0 i$$

$$\therefore B = \frac{N_1 \mu_0 i}{2\pi r}$$

$$\Phi_{B_T} = N_2 \overbrace{\int B \cdot dA}^{\Phi_B \text{ per turn}}$$

$$= N_2 \int \frac{N_1 \mu_0 i}{2\pi r} h \cdot dr$$

$$= \frac{N_1 N_2 \mu_0 i h}{2\pi} \cdot \ln r \Big|_a^b$$

$$= \frac{N_1 N_2 \mu_0 i h}{2\pi} \ln \frac{b}{a}$$

$$M \frac{di}{dt} = \frac{d\Phi_B}{dt}$$

$$M i = \frac{N_1 N_2 \mu_0 i h}{2\pi} \ln \frac{b}{a}$$

$$\therefore M = \underline{\underline{\frac{N_1 N_2 \mu_0 h}{2\pi} \ln \frac{b}{a}}}$$