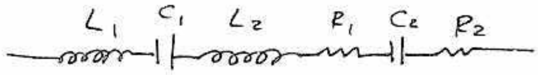
Ch. 31 # 8, 13, 44, 46, 49, 56. 57. 59, 98

# 8




Loop law

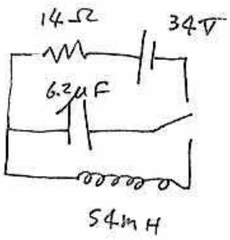
$$+L_1\frac{di}{dt} + \frac{q_1}{C_1} + iR_1 + \frac{q_2}{C_2} + iR_2 + \cdots$$

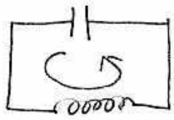
$q_1 = q_2 = \cdots$ for series

$$(L_1 + L_2 + \cdots)\frac{di}{dt} + \left(\frac{q_1}{C_1} + \frac{q_2}{C_2} + \cdots\right) + i(R_1 + R_2 + \cdots)$$

$$L_{eq}\frac{di}{dt} + \frac{q}{C_{eq}} + iR_{eq}$$

# 13.



a)

Loop

$$-\frac{q}{C} - L\frac{di}{dt} = 0$$

$$\frac{1}{C}q + L\frac{di}{dt} = 0 \qquad i = \frac{dq}{dt} = \dot{q}$$

$$\therefore \frac{di}{dt} = \frac{d^2q}{dt^2} = \ddot{q}$$

$$\frac{1}{C}q + L\ddot{q} = 0$$ (This is a simple harmonic oscillation!)

Let
$$q = A\cos(\omega t + \phi)$$
$$\dot{q} = -A\omega\sin(\omega t + \phi)$$
$$\ddot{q} = -A\omega^2\cos(\omega t + \phi)$$

$$\frac{1}{C}A\cos(\omega t + \phi) + L(-A\omega^2\cos(\omega t + \phi)) = 0$$

$$A\cos(\omega t + \phi)\left(\frac{1}{C} - L\omega^2\right) = 0$$

$$\therefore \frac{1}{C} - L\omega^2 = 0$$

$$\omega = \sqrt{\frac{1}{LC}}$$

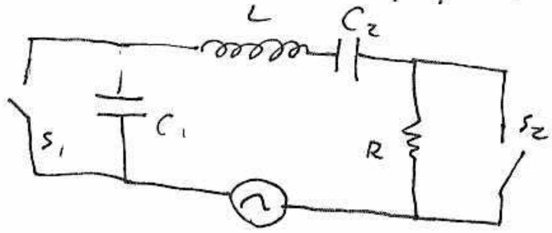
$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = 2.750599799 \times 10^2 \text{ Hz}$$

(b) when $t=0$, $q$ is max ( $= CV = 6.2\times10^{-6}\cdot 34$ )

$$q = CV = A\,\cos(\omega t+\phi) \nearrow 1 = 6.2\times10^{-6}\cdot 34 = 2.108\times10^{-4}\ C$$

$$\therefore\ \dot{q} = \underbrace{|-\omega A|}_{\text{Amp. for } \dot{q}}\ \sin(\omega t+\phi)\nearrow 1 = \underline{0.3643156954\ \text{amp}}$$

# 44

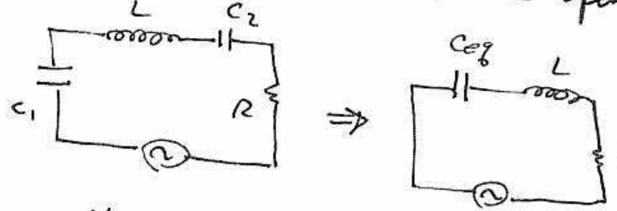



$\mathcal{E}_{max} = 12\,V$

$\nu = 60\ Hz$

$C_1 = C_2$

Case I: when $S_1$ & $S_2$ are open, $i$ leads $\mathcal{E}$ by $30.9°$

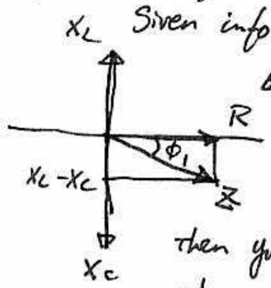



$$C_{eq} = \frac{1}{\frac{1}{C}+\frac{1}{C}} = \frac{1}{2}C$$

$X_L = \omega L$

$X_C = \dfrac{1}{\omega C_{eq}} = \dfrac{2}{\omega C}$

You should be able to construct an impedance diagram from the given info.

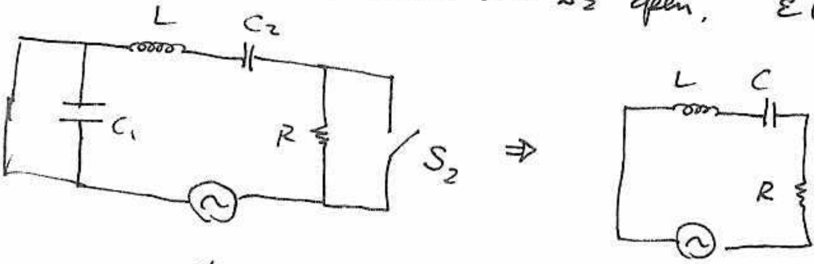



Then you will see why $\phi_1 = -30.9°$

$$\phi_1 = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{\omega L - \frac{2}{\omega C}}{R} = -30.9°$$

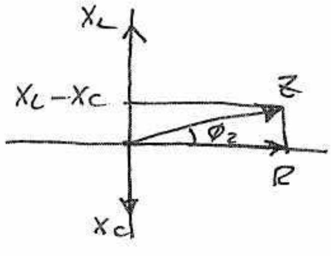
$$\therefore\ \frac{\omega L - \frac{2}{\omega C}}{R} = \tan\phi_1 \quad \text{———— (1)}$$

Case 2: when $S_1$ closed but $S_2$ open, $\mathcal{E}$ leads $i$ by $15°$




$X_L = \omega L$

$X_C = \dfrac{1}{\omega C}$



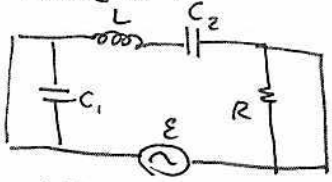

$$\phi_2 = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R} = 15.0°$$

$$\therefore\ \frac{\omega L - \frac{1}{\omega C}}{R} = \tan\phi_2 \quad \text{———— (2)}$$

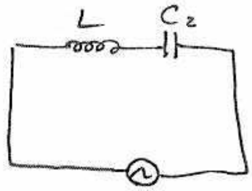
$$Z = \sqrt{(X_L - X_C)^2}$$

$$= (X_L - X_C)$$

Case 3: Both switches closed, $i = 447\ mA$




$\Rightarrow$




$$\frac{\mathcal{E}}{i} = Z = X_L - X_C = 447\ mA$$

$$\frac{\mathcal{E}}{i} = \omega L - \frac{1}{\omega C}$$

$$\omega L = \frac{\mathcal{E}}{i} + \frac{1}{\omega C}$$

$$L = \frac{\frac{\mathcal{E}}{i} + \frac{1}{\omega C}}{\omega} \quad \text{———} \ (3)$$

(2) $\leftarrow$ (3)

$$\frac{\omega\left(\frac{\frac{\mathcal{E}}{i} + \frac{1}{\omega C}}{\omega}\right) - \frac{1}{\omega C}}{R} = \tan 15^\circ$$

$$\frac{\frac{\mathcal{E}}{i} + \frac{1}{\omega C} - \frac{1}{\omega C}}{R} = \tan 15^\circ$$

$$\therefore \quad R = \frac{\frac{\mathcal{E}}{i}}{\tan 15^\circ} = \frac{\frac{12}{447 \times 10^{-3}}}{\tan 15^\circ} = \underline{\underline{100.1892834\ \Omega}} \quad \text{———} \ (2)'$$

(1) $\leftarrow$ (3) & (2)'

$$\frac{\omega\left(\frac{\frac{\mathcal{E}}{i} + \frac{1}{\omega C}}{\omega}\right) - \frac{2}{\omega C}}{R} = \tan(-30.9^\circ)$$

$$\frac{\frac{\mathcal{E}}{i} + \frac{1}{\omega C} - \frac{2}{\omega C}}{R} = \tan(-30.9^\circ)$$

$$\frac{\frac{\mathcal{E}}{i} - \frac{1}{\omega C}}{R} = \tan(-30.9^\circ)$$

$$\frac{\mathcal{E}}{i} - \frac{1}{\omega C} = R \tan(-30.9^\circ) \Rightarrow \quad C = \frac{1}{\omega\left(\frac{\mathcal{E}}{i} - R\tan(-30.9^\circ)\right)} = \frac{1}{2\pi(60)\left(\frac{12}{447 \times 10^{-3}} - R\tan(-30.9^\circ)\right)}$$

$$= \underline{\underline{30.55695886\ \mu F}} \quad \text{———} \ (1)'$$

(3) $\leftarrow$ (1)'

$$L = \frac{\frac{\mathcal{E}}{i} + \frac{1}{\omega C}}{\omega} = \frac{\frac{12}{447 \times 10^{-3}} + \frac{1}{2\pi \nu C}}{2\pi \nu} = \underline{\underline{301.474806\ mH}}$$

#46

$\mathcal{E}_m = 220\,V$

$\nu = 400\,Hz$

$R = 220\,\Omega$

$L = 150 \times 10^{-3}\,H$

$C = 24 \times 10^{-6}\,F$

(a) $X_C = \frac{1}{\omega C} = \underline{\underline{16.57863991\,\Omega}}$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{220^2 + (\underbrace{2\pi(400) \cdot 150 \times 10^{-3} - 16.5786\cdots}_{\text{Notice : } X_L > X_C})^2}$

$= \underline{\underline{422.2524774\,\Omega}}$

(c) $i_{max} = \frac{\mathcal{E}_m}{Z} = \underline{\underline{0.5210152972 \text{ amp}}}$

(d)
(e)
(f)

$C_{eq} = \frac{1}{\frac{1}{C} + \frac{1}{C}} = \underline{12\,\mu F}$

$C_{eq}$ is a half of the original

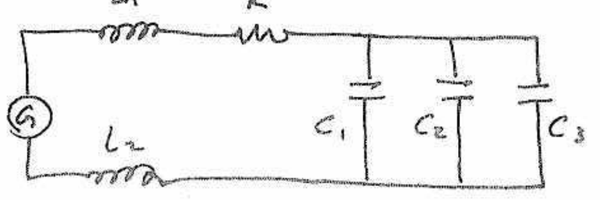
$\rightarrow X_C = \frac{1}{\omega C} \rightarrow$ $\underline{\underline{\text{Doubled}}}$

$\rightarrow$ Since $X_L > X_C$ before, $(X_L - X_C)$ is less, then $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is less.

$\rightarrow$ $\underline{\underline{Z \text{ should decrease}}}$

$\rightarrow$ then $\underline{\underline{i \text{ should increase}}}$

#49




$R = 100\,\Omega$

$L_1 = 1.7 \times 10^{-3}\,H$

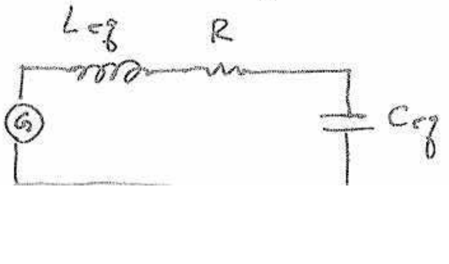
$L_2 = 2.3 \times 10^{-3}\,H$

$C_1 = 4 \times 10^{-6}\,F$

$C_2 = 2.5 \times 10^{-6}\,F$

$C_3 = 3.5 \times 10^{-6}\,F$

$\Downarrow$



(a) $L_{eq} = L_1 + L_2$ ( Series ) $= 4\times10^{-3}\,H$

$C_{eq} = C_1 + C_2 + C_3$ ( parallel ) $= 10\times10^{-6}\,F$

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{4\times10^{-3}\cdot 10\times10^{-6}}} = 5000 \text{ rad/sec}$$

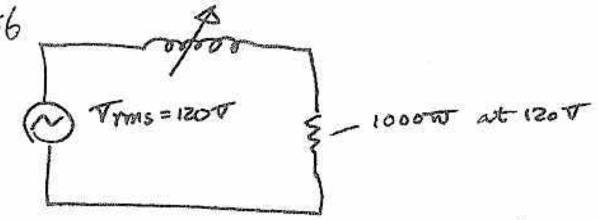
$$\omega = 2\pi\nu \rightarrow \nu = \frac{\omega}{2\pi} = \frac{5000}{2\pi} = \underline{\underline{795.7747155\,Hz}}$$

(b) Since $\omega_R = \sqrt{\frac{1}{LC}}$ is independent from R,
resonant freq. will not change.

(c) If $L_1$ increases, $\omega_R$ decreases $\rightarrow$ $\underline{\underline{\nu_{res.} \text{ also decreases.}}}$

(d) If $C_3$ is gone, $C_{eq}$ decreases $\rightarrow$ $\omega_R$ increases & $\underline{\underline{\nu_{res} \text{ increases}}}$

#56




$V = IR \rightarrow I = \frac{V}{R}$

$P = I\cdot V \longrightarrow P = \frac{V^2}{R} \quad \therefore R = \frac{V^2}{P}$

$= \frac{(120)^2}{1000}$

$= \underline{\underline{14.4\,\Omega}}$

(a)
$$\frac{P_{max}}{P_{min}} = 5 = \frac{i_{max}^2\,R}{i_{min}^2\,R} = \frac{\left(\frac{V_{rms}}{Z_{min}}\right)^2\cdot R}{\left(\frac{V_{rms}}{Z_{max}}\right)^2\cdot R} = \frac{Z_{max}^2}{Z_{min}^2}$$

$$= \frac{R^2 + (\omega L)^2}{R^2}$$

$$\therefore 5R^2 = R^2 + (\omega L)^2$$

$$L^2 = \frac{4R^2}{\omega^2} = \frac{4R^2}{(2\pi\nu)^2} = \frac{4(14.4)^2}{4\pi^2\cdot 60^2} = \underline{\underline{7.639437268\times10^{-2}\,H}}$$

(b), (c)

modifying $5 = \frac{R^2 + (\omega L)^2}{R^2}$ from (a),

$$5 = \frac{(R_{Light} + R)^2}{R_{Light}^2}$$

$$5R_{Light}^2 = (R_{Light} + R)^2$$

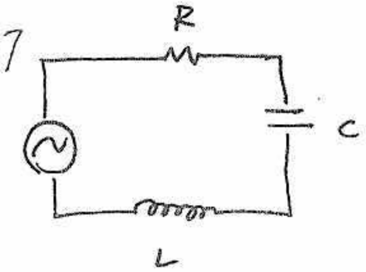
$$\sqrt{5}\,R_{Light} = R_{Light} + R$$

$\therefore R = (\sqrt{5}-1) R_{Light}$

$= (\sqrt{5}-1)(14.4) = \underline{\underline{17.79937888\ \Omega}}$ yes, it can be done

(d) In (a), L stores energy as magnetic energy (which is retractable)
In (b), R uses energy and it can not be retracted.

#57




$R = 5\Omega$

$L = 60 \times 10^{-3} H$

$f_d = 60\ Hz$

$\mathcal{E}_m = 30\ V$

$i_{rms} = \frac{V_{rms}}{Z}$

(a) $\overline{P} = (i_{rms})^2 \cdot R$

We need to minimize $Z$, $\rightarrow \sqrt{R^2 + (X_L - X_C)^2}$ minimized.

$\rightarrow X_L - X_C = 0 \rightarrow \omega L - \frac{1}{\omega C} = 0$

$\therefore C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 \cdot L} = \underline{\underline{1.172698885 \times 10^{-4} F}}$

(b) To minimize P, $i$ should be minimized $\rightarrow Z = \infty \rightarrow X_C \rightarrow \infty$

$\rightarrow \frac{1}{\omega C} = \infty \rightarrow \underline{\underline{C = 0}}$

(c) $P_{max} = (i^{rms}_{max})^2 \cdot R$ (when $Z$ is minimum)

$= \left(\frac{V_{max}/\sqrt{2}}{R}\right)^2 \cdot R = \frac{V_{max}}{2R} = \underline{\underline{90\ W}}$

(f) $P_{min} = 0 \cdot R = \underline{\underline{0\ W}}$

$P_{max}$ happens when $i = i_{max}$.

(d) $i_{max}$ happens when $Z$ is min $\rightarrow X_L - X_C = 0 \Rightarrow \tan^{-1}\frac{X_L - X_C}{R} = \underline{\underline{0}}$

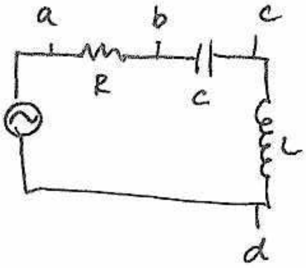
$P_{min}$ happens when $i = 0$ (No shift!)

(g) $i = 0$ when $Z$ is $\infty$ $\rightarrow X_L - X_C = -\infty$ (because $C = 0$) $\rightarrow \tan^{-1}\frac{X_L - X_C}{R} = \underline{\underline{-\frac{\pi}{2}}}$

(e)
(h) Power factor : $\cos\phi = 1$ for max (I & V are in sync.)

$\cos\phi = 0$ for min (I & V are totally off-sync.)

#59




$R = 15\,\Omega$

$C = 4.7\times10^{-6}\,F$

$L = 25\times10^{-3}\,H$

$V_{rms} = 75\,V$

$\nu = 550\,Hz \Rightarrow \omega = 2\pi\nu$

(a)
$$i_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{V_{rms}}{\sqrt{R^2+\left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}} = \underline{\underline{2.585763724\ \text{amp}}}$$

(b)
$V_{ab\,rms} = iR = \underline{\underline{38.78645587\,V}}$

$V_{bc\,rms} = iX_C = i\left(\frac{1}{\omega C}\right) = \underline{\underline{159.2019746\,V}}$

$V_{cd\,rms} = iX_L = i\omega L = \underline{\underline{223.3939488\,V}}$

$V_{ad\,rms} = V_{rms} = 75\,V$

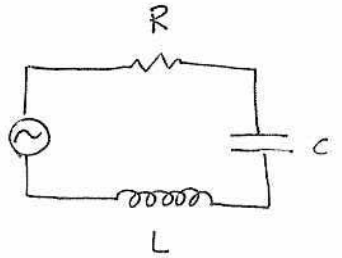
The difference between this (rms) & #42 is either average (rms) or at any given instance. Make sure you can do both.

(c)
$P_R = i^2R = 100.2926106\,W$

$P_C = 0$

$P_L = 0$ } they store energy, they do not disapate energy.

#98




$R = 200\,\Omega$

$C = 15\,\mu F$

$L = 230\,mH$

$f_d = 60\,Hz$

$\mathcal{E}_m = 36.0\,V$

(a) $\mathcal{E}_{max} = 36.0\,V$ (Given)

Before we solve for the rest, we need some work

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= 217.3708587\ \Omega$

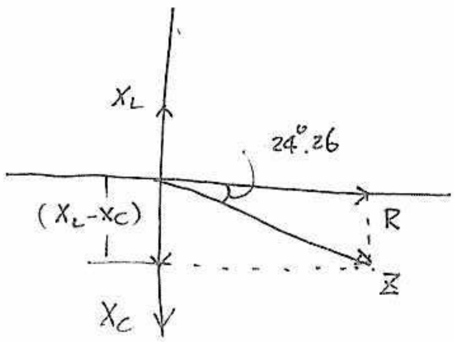
$X_L = \omega L = 2\pi \nu L = 87.70775724\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = 176.8388257\ \Omega$

$\therefore\ i_{max} = \frac{\mathcal{E}_{max}}{Z} = 0.1641056624$ amp

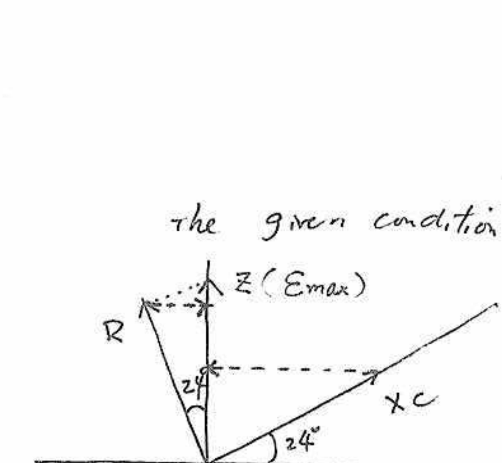
$\phi = \tan^{-1} \frac{X_L - X_C}{R} = -24^\circ.2589152 7$

Here is the Key point to understand LCR circuit.




It's very important to draw a phasor diagram as accurately as possible.
This diagram shows that
1. the current (same direction as R) is 24°.26 ahead of V (same direction as Z)
2. y- projections show values of what we are looking for. (In this instant, $i = 0$ and $V_{net}$ = negative, $V_L$ = its max, $V_C$ = its min, and $V_R = 0$.)

The given condition is that $\mathcal{E}$ = max.




So $\mathcal{E}$ direction is on y axis and others follow.
At this moment V across each part is
1. Calculate $V_{max}$ of each.
2. project that value on to the y-axis.

Remember that $\vec{R}$, $\vec{X_C}$ & $\vec{X_L}$ are <u>always</u> rotating.
These solutions are true only when $\mathcal{E} = \mathcal{E}_{max}$ (when $\vec{\mathcal{E}}$ is on y axis)

(b) $V_R = i_{max} R \cos 24^\circ... = 29.92296485$ V

(c) $V_C = i_{max} X_C \sin 24^\circ... = 11.9238182$ V

(d) $V_L = i_{max} X_L (-\sin 24^\circ...) = -5.913671321$ V (see the diagram. $V_L$ is down)

(e) +

$35.9331147 3\ \text{V} \sim 36\ \text{V}$