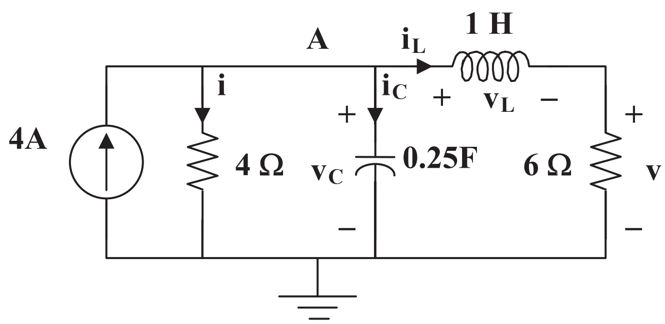
Chapter 8, Solution 5.

(a) For $t < 0$, $4u(t) = 0$ so that the circuit is not active (all initial conditions = 0).

$$i_L(0-) = 0 \text{ and } v_C(0-) = 0.$$

For $t = 0+$, $4u(t) = 4$. Consider the circuit below.



Since the 4-ohm resistor is in parallel with the capacitor,

$$i(0+) = v_C(0+)/4 = 0/4 = \mathbf{0\ A}$$

Also, since the 6-ohm resistor is in series with the inductor,
$v(0+) = 6i_L(0+) = $ **0V**.

(b) $di(0+)/dt = d(v_R(0+)/R)/dt = (1/R)dv_R(0+)/dt = (1/R)dv_C(0+)/dt$

$$= (1/4)4/0.25 \text{ A/s} = \mathbf{4\ A/s}$$

$v = 6i_L$ or $dv/dt = 6di_L/dt$ and $dv(0+)/dt = 6di_L(0+)/dt = 6v_L(0+)/L = 0$

$$\text{Therefore } dv(0+)/dt = \mathbf{0\ V/s}$$

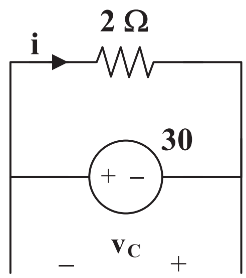
(c) As t approaches infinity, the circuit is in steady-state.

$$i(\infty) = 6(4)/10 = \mathbf{2.4\ A}$$

$$v(\infty) = 6(4 - 2.4) = \mathbf{9.6\ V}$$

Chapter 8, Solution 20.

For $t < 0$, the equivalent circuit is as shown below.



$$v(0) = -30 \text{ V and } i(0) = 30/2 = 15 \text{ A}$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2x0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5x1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

$$i(t) = (A\cos(2t) + B\sin(2t))e^{-2t}$$

$$i(0) = 15 = A$$

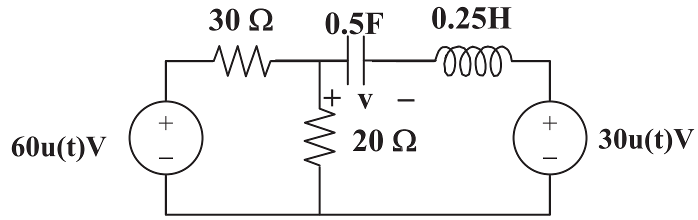
$$di/dt = -2(15\cos(2t) + B\sin(2t))e^{-2t} + (-2x15\sin(2t) + 2B\cos(2t))e^{-\alpha t}$$

$$di(0)/dt = -30 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[30 - 30] = 0$$

Thus, $B = 15$ and $i(t) =$ $\mathbf{(15\cos(2t) + 15\sin(2t))e^{-2t}\ A}$

Chapter 8, Solution 39.

For $t = 0^-$, the source voltages are equal to zero thus, the initial conditions are v(0) = 0 and $i_L(0) = 0$.



For $t > 0$, the circuit is shown above.

$$R = 20||30 = 12 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/4)} = \sqrt{8}$$

$$\alpha = R/(2L) = (12)/(0.5) = 24$$

Since $\alpha > \omega_o$, we have an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -47.83,\ -0.167$$

Thus, $v(t) = V_s + [Ae^{-47.83t} + Be^{-0.167t}]$, where
$V_s = [60/(30+20)]20-30 = -6$ volts.

$$v(0) = 0 = -6 + A + B \text{ or } 6 = A + B \qquad (1)$$

$$i(0) = Cdv(0)/dt = 0$$

But, $dv(0)/dt = -47.83A - 0.167B = 0$ or

$$B = -286.4A \qquad (2)$$

From (1) and (2), $A + (-286.4)A = 6$ or $A = 6/(-285.4) = -0.02102$ and
$B = -286.4x(-0.02102) = 6.02$

$$v(t) = \mathbf{[-6 + (-0.021e^{-47.83t} + 6.02e^{-0.167t})]\ volts}.$$

Chapter 8, Solution 43.

For t>0, we have a source-free series RLC circuit.

$$\alpha = \frac{R}{2L} \longrightarrow R = 2\alpha L = 2x8x0.5 = \mathbf{8\Omega}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 30 \longrightarrow \omega_o = \sqrt{900 + 64} = \sqrt{964}$$

$$\boldsymbol{\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_o^2 L} = \frac{1}{964x0.5} = 2.075\ \text{mF}}$$

Chapter 8, Solution 49.

$$\text{For } t = 0^-,\ i(0) = 3 + 12/4 = 6 \text{ and } v(0) = 0.$$

For $t > 0$, we have a parallel *RLC* circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus,
$$i(t) = I_s + [(A + Bt)e^{-2t}],\quad I_s = 3$$

$$i(0) = 6 = 3 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

Thus, $i(t) = \mathbf{\{3 + [(3 + 6t)e^{-2t}]\}\ A}$

Chapter 8, Solution 58.

(a) Let i =inductor current, v = capacitor voltage i(0) =0, v(0) = 4

$$\frac{dv(0)}{dt} = -\frac{[v(0) + Ri(0)]}{RC} = -\frac{(4+0)}{0.5} = \underline{-8\ \text{V/s}}$$

(b) For $t \geq 0$, the circuit is a source-free RLC parallel circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2x0.5x1} = 1, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2$$

$$\omega_d = \sqrt{\omega^2{}_o - \alpha^2} = \sqrt{4-1} = 1.732$$

Thus,

$$v(t) = e^{-t}(A_1 \cos 1.732t + A_2 \sin 1.732t)$$
$$v(0) = 4 = A_1$$

$$\frac{dv}{dt} = -e^{-t} A_1 \cos 1.732t - 1.732 e^{-t} A_1 \sin 1.732t - e^{-t} A_2 \sin 1.732t + 1.732 e^{-t} A_2 \cos 1.732t$$

$$\frac{dv(0)}{dt} = -8 = -A_1 + 1.732 A_2 \quad \longrightarrow \quad A_2 = -2.309$$

$$\boldsymbol{v(t) = \underline{e^{-t}(4\cos 1.732t - 2.309 \sin 1.732t)}\ \text{V}}$$

Chapter 8, Solution 60.

At $t = 0-$, $4u(t) = 0$ so that $i_1(0) = 0 = i_2(0)$ (1)

Applying nodal analysis,

$4 = 0.5di_1/dt + i_1 + i_2$
(2)

Also, $i_2 = [1di_1/dt - 1di_2/dt]/3$ or $3i_2 = di_1/dt - di_2/dt$ (3)

Taking the derivative of (2), $0 = d^2i_1/dt^2 + 2di_1/dt + 2di_2/dt$ (4)

From (2) and (3), $di_2/dt = di_1/dt - 3i_2 = di_1/dt - 3(4 - i_1 - 0.5di_1/dt)$

$= di_1/dt - 12 + 3i_1 + 1.5di_1/dt$

Substituting this into (4),

$d^2i_1/dt^2 + 7di_1/dt + 6i_1 = 24$ which gives $s^2 + 7s + 6 = 0 = (s + 1)(s + 6)$

Thus, $i_1(t) = I_s + [Ae^{-t} + Be^{-6t}]$, $6I_s = 24$ or $I_s = 4$

$i_1(t) = 4 + [Ae^{-t} + Be^{-6t}]$ and $i_1(0) = 4 + [A + B]$ (5)

$i_2 = 4 - i_1 - 0.5di_1/dt = i_1(t) = 4 + -4 - [Ae^{-t} + Be^{-6t}] - [-Ae^{-t} - 6Be^{-6t}]$

$= [-0.5Ae^{-t} + 2Be^{-6t}]$ and $i_2(0) = 0 = -0.5A + 2B$ (6)

From (5) and (6), $A = -3.2$ and $B = -0.8$

$i_1(t) =$ **$\{4 + [-3.2e^{-t} - 0.8e^{-6t}]\}$ A**

$i_2(t) =$ **$[1.6e^{-t} - 1.6e^{-6t}]$ A**

Chapter 8, Solution 63.

$$\frac{v_s - 0}{R} = C\frac{d(0 - v_o)}{dt} \longrightarrow \frac{v_s}{R} = -C\frac{dv_o}{dt}$$

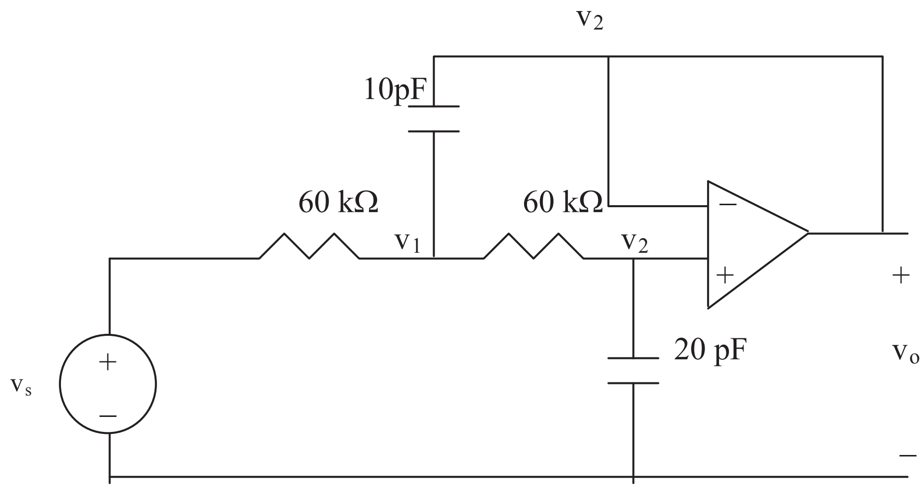
$$v_o = L\frac{di}{dt} \longrightarrow \frac{dv_o}{dt} = L\frac{d^2 i}{dt^2} = -\frac{v_s}{RC}$$

Thus,

$$\boldsymbol{\frac{d^2 i(t)}{dt^2} = -\frac{v_s}{RCL}}$$

Chapter 8, Solution 66.

We apply nodal analysis to the circuit as shown below.



At node 1,

$$\frac{v_s - v_1}{60k} = \frac{v_1 - v_2}{60k} + 10pF\frac{d}{dt}(v_1 - v_o)$$

But $v_2 = v_o$

$$v_s = 2v_1 - v_o + 6x10^{-7}\frac{d(v_1 - v_o)}{dt} \qquad (1)$$

At node 2,

$$\frac{v_1 - v_2}{60k} = 20pF\frac{d}{dt}(v_2 - 0), \qquad v_2 = v_o$$

$$v_1 = v_o + 1.2x10^{-6}\frac{dv_o}{dt} \qquad (2)$$

Substituting (2) into (1) gives

$$v_s = 2\left(v_o + 1.2x10^{-6}\frac{dv_o}{dt}\right) - v_o + 6x10^{-7}\left(1.2x10^{-6}\frac{d^2v_o}{dt^2}\right)$$

$$v_s = \mathbf{v_o + 2.4x10^{-6}(dv_o/dt) + 7.2x10^{-13}(d^2v_o/dt^2)}.$$

Chapter 9, Solution 2.

(a) amplitude = **15 A**

(b) $\omega = 25\pi =$ **78.54 rad/s**

(c) $f = \dfrac{\omega}{2\pi} =$ **12.5Hz**

(d) $I_s = 15\angle 25° \text{ A}$

$$I_s(2 \text{ ms}) = 15\cos((500\pi)(2\times 10^{-3}) + 25°)$$
$$= 15\cos(\pi + 25°) = 15\cos(205°)$$
$$= \mathbf{-13.595\ A}$$

Chapter 9, Solution 8.

(a) $\dfrac{60\angle 45°}{7.5 - j10} + j2 = \dfrac{60\angle 45°}{12.5\angle -53.13°} + j2$

$= 4.8\angle 98.13° + j2 = -0.6788 + j4.752 + j2$

$= \mathbf{-0.6788 + j6.752}$

(b) $(6-j8)(4+j2) = 24-j32+j12+16 = 40-j20 = 44.72\angle -26.57°$

$\dfrac{32\angle -20°}{(6-j8)(4+j2)} + \dfrac{20}{-10+j24} = \dfrac{32\angle -20°}{44.72\angle -26.57°} + \dfrac{20}{26\angle 112.62°}$

$= 0.7156\angle 6.57° + 0.7692\angle -112.62° = 0.7109 + j0.08188 - 0.2958 - j0.71$

$= \mathbf{0.4151 - j0.6281}$

(c) $20 + (16\angle -50°)(13\angle 67.38°) = 20 + 208\angle 17.38° = 20 + 198.5 + j62.13$

$= \mathbf{218.5 + j62.13}$

Chapter 9, Solution 31.

$$L = 240mH \quad \longrightarrow \quad j\omega L = j2x240x10^{-3} = j0.48$$

$$C = 5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2x5x10^{-3}} = -j100$$

$$Z = 80 + j0.48 - j100 = 80 - j99.52 =$$

$$I = \frac{V}{Z} = \frac{10 < 0^0}{80 - j99.52} = 0.0783 < 51.206^o$$

i(t) = **78.3cos(2t+51.21°) mA**

Chapter 9, Solution 37.

$$Y = (1/4) + (1/(j8)) + (1/(-j10)) = 0.25 - j0.025$$

$$= \mathbf{(250 - j25)\ mS}$$

Chapter 9, Solution 39.

$$Z_{eq} = 4 + j20 + 10 // (-j14 + j25) = \underline{9.135 + j27.47 \ \ \Omega}$$

= **(9.135+j27.47) Ω**

$$I = \frac{V}{Z_{eq}} = \frac{12}{9.135 + j27.47} = 0.4145 < -71.605^o$$

i(t) = **414.5cos(10t–71.6˚) mA**

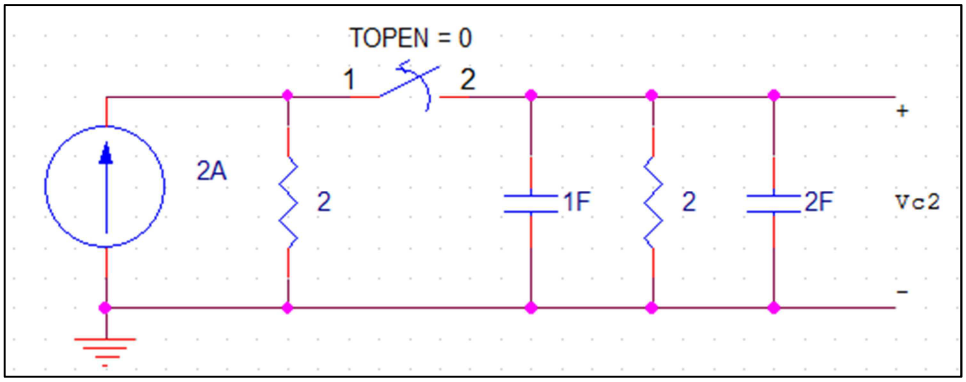
Chapter 9, Solution 60.

$$Z = (25 + j15) + (20 - j50)//(30 + j10) = 25 + j15 + 26.097 - j5.122$$

Z = **(51.1+j9.878) Ω**

Supplemental Question

MODEL:



a. Determine the equation that represents "Vc2"

<u>INITIAL CONDITIONS</u>

$Vc(0-) = Vc(0+) = 2V$

$Vc(\infty) = 0V$

<u>CIRCUIT EQUATION</u>

$C1\ dVc/dt + Vc / R + C2\ dVc/dt = 0$

$dVc/dt\ (C1 + C2) + Vc/2 = 0$

$\tau = (C1 + C2) * R = 6$ Seconds

<u>GENERAL SOLUTION</u>

$Vc(t) = Vh + Vp$

$Vh = Ke^{-t/\tau}$

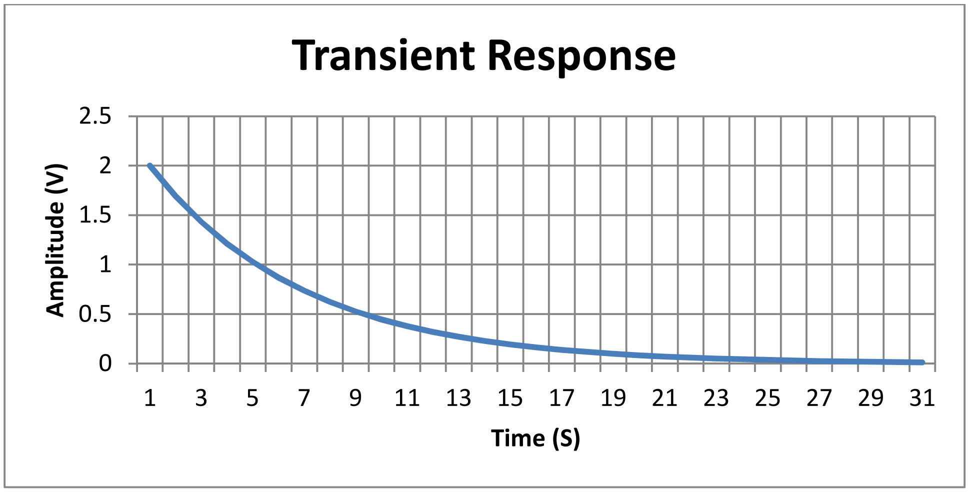
$Vc(0+) = 2 = Ke^{0} + Vp$

➔ $K = 2 - Vp$

$Vp = Vc(\infty) = 0$

$Vc(t) = 2e^{-t/\tau}$

b. Sketch the voltage characteristics



Transient Response
Amplitude (V)
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
Time (S)