Chapter 11, Solution 7.

Applying KVL to the left-hand side of the circuit,

$$8\angle 20° = 4\mathbf{I}_o + 0.1\mathbf{V}_o \qquad (1)$$

Applying KCL to the right side of the circuit,

$$8\mathbf{I}_o + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1}{10 - j5} = 0$$

But, $$\mathbf{V}_o = \frac{10}{10 - j5}\mathbf{V}_1 \longrightarrow \mathbf{V}_1 = \frac{10 - j5}{10}\mathbf{V}_o$$

Hence, $$8\mathbf{I}_o + \frac{10 - j5}{j50}\mathbf{V}_o + \frac{\mathbf{V}_o}{10} = 0$$

$$\mathbf{I}_o = j0.025\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1),

$$8\angle 20° = 0.1\mathbf{V}_o(1 + j)$$

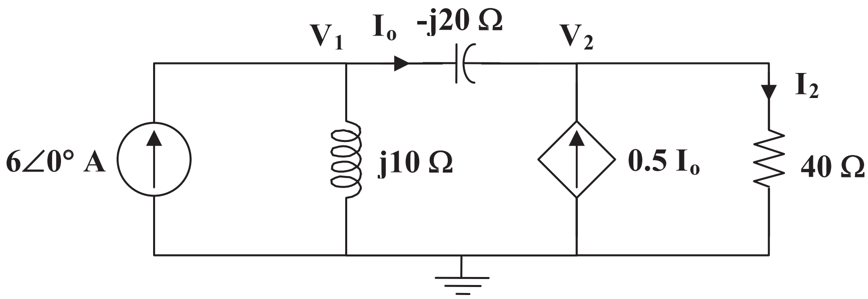
$$\mathbf{V}_o = \frac{80\angle 20°}{1 + j}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{10} = \frac{8}{\sqrt{2}}\angle -25°$$

$$P = \frac{1}{2}|\mathbf{I}_1|^2 R = \left(\frac{1}{2}\right)\left(\frac{64}{2}\right)(10) = \mathbf{160W}$$

Chapter 11, Solution 8.

We apply nodal analysis to the following circuit.



At node 1,

$$6 = \frac{\mathbf{V}_1}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} \quad \mathbf{V}_1 = j120 - \mathbf{V}_2 \tag{1}$$

At node 2,

$$0.5\mathbf{I}_o + \mathbf{I}_o = \frac{\mathbf{V}_2}{40}$$

But, $\mathbf{I}_o = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$

Hence, $\dfrac{1.5(\mathbf{V}_1 - \mathbf{V}_2)}{-j20} = \dfrac{\mathbf{V}_2}{40}$

$$3\mathbf{V}_1 = (3 - j)\mathbf{V}_2 \tag{2}$$

Substituting (1) into (2),

$$j360 - 3\mathbf{V}_2 - 3\mathbf{V}_2 + j\mathbf{V}_2 = 0$$

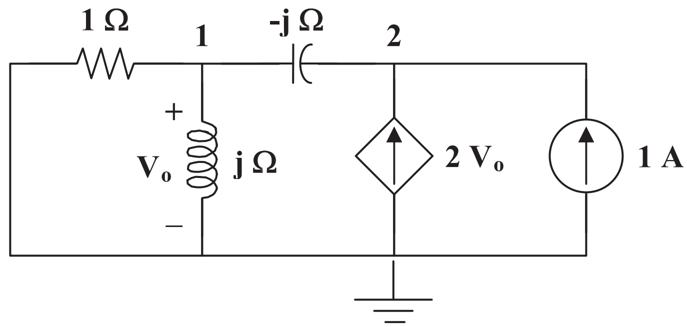
$$\mathbf{V}_2 = \frac{j360}{6 - j} = \frac{360}{37}(-1 + j6)$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{40} = \frac{9}{37}(-1 + j6)$$

$$P = \frac{1}{2}\left|\mathbf{I}_2\right|^2 R = \frac{1}{2}\left(\frac{9}{\sqrt{37}}\right)^2 (40) = \mathbf{43.78\ W}$$

Chapter 11, Solution 15.

To find $\mathbf{Z}_{eq}$, insert a 1-A current source at the load terminals as shown in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_o}{1}+\frac{\mathbf{V}_o}{j}=\frac{\mathbf{V}_2-\mathbf{V}_o}{-j}\longrightarrow \mathbf{V}_o=j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$1+2\mathbf{V}_o=\frac{\mathbf{V}_2-\mathbf{V}_o}{-j}\longrightarrow 1=j\mathbf{V}_2-(2+j)\mathbf{V}_o \qquad (2)$$

Substituting (1) into (2),

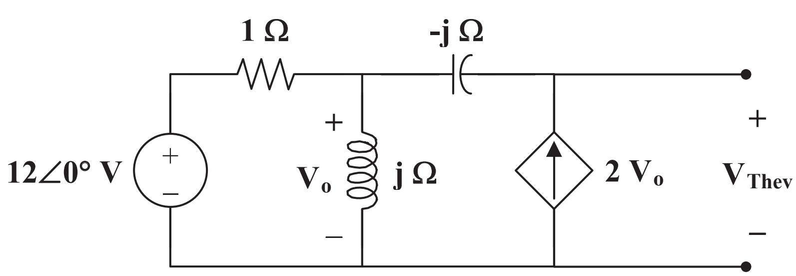
$$1=j\mathbf{V}_2-(2+j)(j)\mathbf{V}_2=(1-j)\mathbf{V}_2$$

$$\mathbf{V}_2=\frac{1}{1-j}$$

$$\mathbf{Z}_{eq}=\frac{\mathbf{V}_2}{1}=\frac{1+j}{2}=0.5+j0.5$$

$$\mathbf{Z}_L=\mathbf{Z}_{eq}^{*}=\mathbf{[0.5-}\boldsymbol{j}\mathbf{0.5]\,\Omega}$$

We now obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

$$-2\mathbf{V}_o+\frac{\mathbf{V}_o-12}{1}+\frac{\mathbf{V}_o}{j}=0$$

$$\mathbf{V}_o = \frac{-12}{1+j}$$

$$-\mathbf{V}_o - (-j \times 2\,\mathbf{V}_o) + \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Thev} = (1 - j2)\mathbf{V}_o = \frac{(-12)(1-j2)}{1+j}$$

$$P_{max} = \frac{\left\| \left[\dfrac{V_{Thev}}{0.5 + j0.5 + 0.5 - j0.5} \right] \right\|^2}{2} 0.5 = \frac{\left(\dfrac{12\sqrt{5}}{\sqrt{2}} \right)^2}{2\left(2x0.5\right)^2} 0.5$$

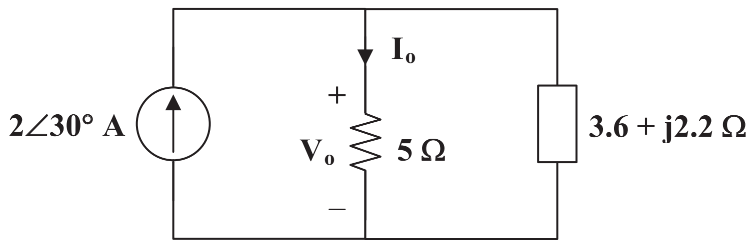
$$= \mathbf{90\ W}$$

Chapter 11, Solution 56.

$$-j2 \,\|\, 6 = \frac{(6)(-j2)}{6-j2} = \frac{12\angle -90^\circ}{6.32456\angle -18.435^\circ} = 1.897365\angle -71.565^\circ = 0.6 - j1.8$$

$$3 + j4 + [(-j2) \,\|\, 6] = 3.6 + j2.2$$

The circuit is reduced to that shown below.



$$\mathbf{I}_o = \frac{3.6 + j2.2}{8.6 + j2.2}(2\angle 30^\circ) = \frac{4.219\angle 31.4296^\circ}{8.87694\angle 14.3493^\circ}(2\angle 30^\circ) = 0.95055\angle 47.08^\circ$$

$$\mathbf{V}_o = 5\mathbf{I}_o = 4.75275\angle 47.08^\circ$$

$$\mathbf{S} = \mathbf{V}_o \mathbf{I}_s^* = (4.75275\angle 47.08^\circ)(2\angle -30^\circ)$$

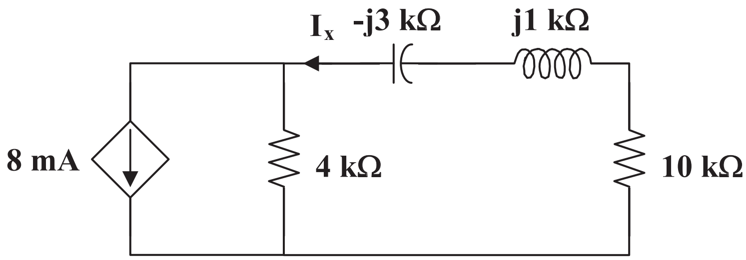
$$\mathbf{S} = 9.5055\angle 17.08^\circ = \mathbf{(9.086+j2.792)\ VA}$$

Chapter 11, Solution 58.

From the left portion of the circuit,

$$\mathbf{I}_o = \frac{0.2}{500} = 0.4 \text{ mA}$$

$20\mathbf{I}_o = 8$ mA which then leads to the following circuit,



From the right portion of the circuit,

$$\mathbf{I}_x = \frac{4}{4+10+j-j3}(8 \text{ mA}) = \frac{16}{7-j} \text{ mA}$$

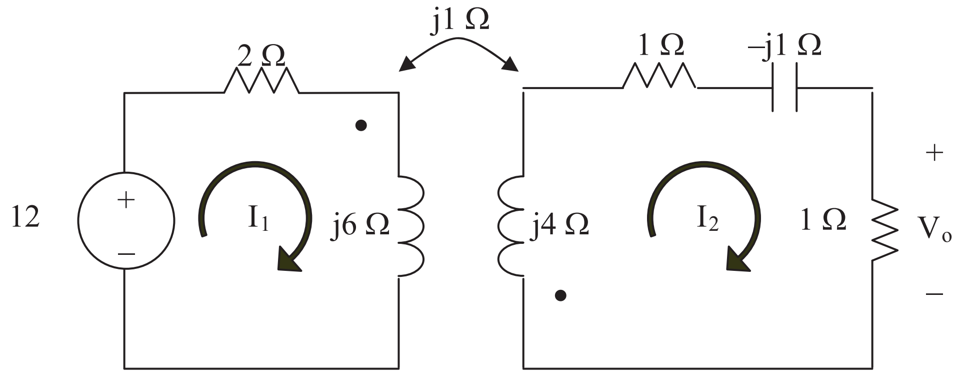
$$\mathbf{S} = |\mathbf{I}_x|^2 R = \frac{(16\times 10^{-3})^2}{50} \cdot (10 \times 10^3)$$

$$\mathbf{S} = \mathbf{51.2\ mVA}$$

It should be noted that the complex power delivered to a resistor is always watts.

Chapter 13, Solution 7.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$(2+j6)I_1 + jI_2 = 24$$

For mesh 2,

$$jI_1 + (2–j+j4)I_2 = jI_1 + (2+j3)I_2 = 0 \text{ or } I_1 = (–3+j2)I_2$$

Substituting into the first equation results in $I_2 = (–0.8762+j0.6328)$ A.

$$V_o = I_2 x1 = \mathbf{1.081\angle 144.16^\circ\ V}.$$

Chapter 13, Solution 17.

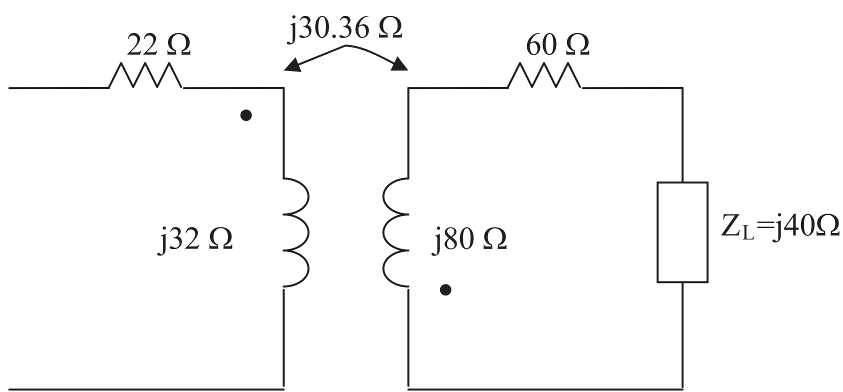
$j\omega L = j40 \quad \longrightarrow \quad \omega = \frac{40}{L} = \frac{40}{15x10^{-3}} = \quad 2667 \text{ rad/s}$

$M = k\sqrt{L_1 L_2} = 0.6\sqrt{12x10^{-3}x30x10^{-3}} = \quad 11.384 \text{ mH}$

If 15 mH ⟶ 40 Ω

Then 12 mH ⟶ 32 Ω
30 mH ⟶ 80 Ω
11.384 mH ⟶ 30.36 Ω

The circuit becomes that shown below.



$$Z_{in} = 22 + j32 + \frac{\omega^2 M^2}{j80 + 60 + j40} = 22 + j32 + \frac{(30.36)^2}{60 + j120}$$

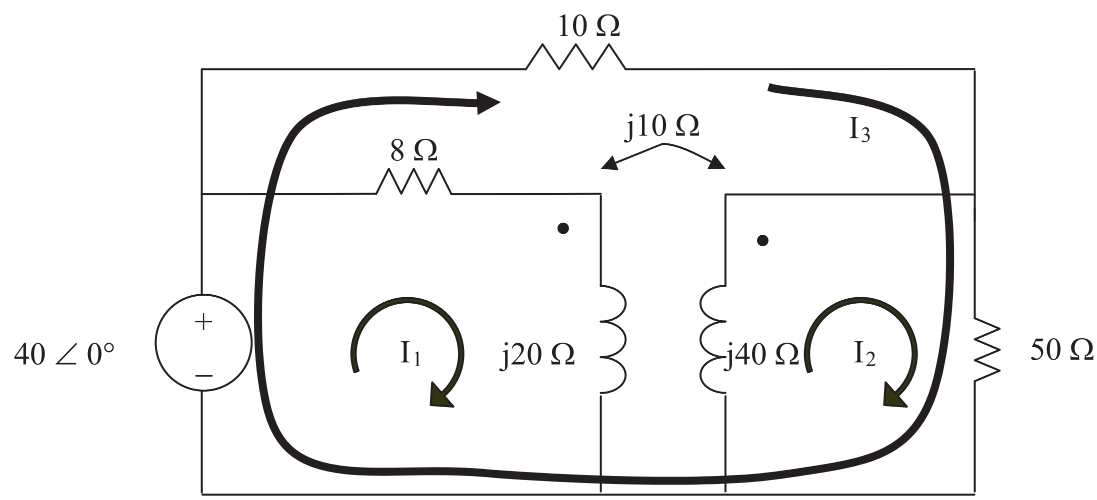
$$= 22 + j32 + \frac{921.7}{134.16\angle 63.43°} = 22 + j32 + 6.87\angle -63.43° = 22 + j32 + 3.073 - j6.144$$

$= \textbf{[25.07 + j25.86]}\ \boldsymbol{\Omega}.$

Chapter 13, Solution 27.

$$1H \longrightarrow j\omega L = j20$$
$$2H \longrightarrow j\omega L = j40$$
$$0.5H \longrightarrow j\omega L = j10$$

We apply mesh analysis to the circuit as shown below.



To make the problem easier to solve, let us have I_3 flow around the outside loop as shown.

For mesh 1,

$$-40 + 8I_1 + j20I_1 - j10I_2 = 0 \text{ or } (8+j20)I_1 - j10I_2 = 40 \quad (1)$$

For mesh 2,

$$j40I_2 - j10I_1 + 50(I_2 + I_3) = 0 \text{ or } -j10I_1 + (50+j40)I_2 + 50I_3 = 0 \quad (2)$$

For mesh 3,

$$-40 + 10I_3 + 50(I_3 + I_2) = 0 \text{ or } 50I_2 + 60I_3 = 40 \quad (3)$$

In matrix form, (1) to (3) become

$$\begin{bmatrix} 8+j20 & -j10 & 0 \\ -j10 & 50+j40 & 50 \\ 0 & 50 & 60 \end{bmatrix} I = \begin{bmatrix} 40 \\ 0 \\ 40 \end{bmatrix}$$

```
>> Z=[(8+20i),-10i,0;-10i,(50+40i),50;0,50,60]
Z =
   8.0000 +20.0000i        0 -10.0000i        0
```

```
    0 -10.0000i  50.0000 +40.0000i  50.0000
    0            50.0000            60.0000

>> V=[40;0;40]
V =
   40
    0
   40

>> I=inv(Z)*V
I =
   0.6354 - 1.5118i
   0.0613 + 0.4682i
   0.6156 - 0.3901i
```

Solving this leads to $\mathbf{I_{50}} = \mathbf{I_2} + \mathbf{I_3}$ = 0.0613+0.6156 + j(0.4682–0.3901) =

0.6769+j0.0781 = 0.6814∠6.58° A or I_{50rms} $|\mathbf{I_{50rms}}|$ = 0.6814/1.4142 = 481.8 mA.

The power delivered to the 50-Ω resistor is

$$P = (I_{50rms})^2R = (0.4818)^2 50 = \mathbf{11.608\ W}.$$

Chapter 13, Solution 36.

Following the two rules in section 13.5, we obtain the following:

(a) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$ $(n = V_2/V_1)$

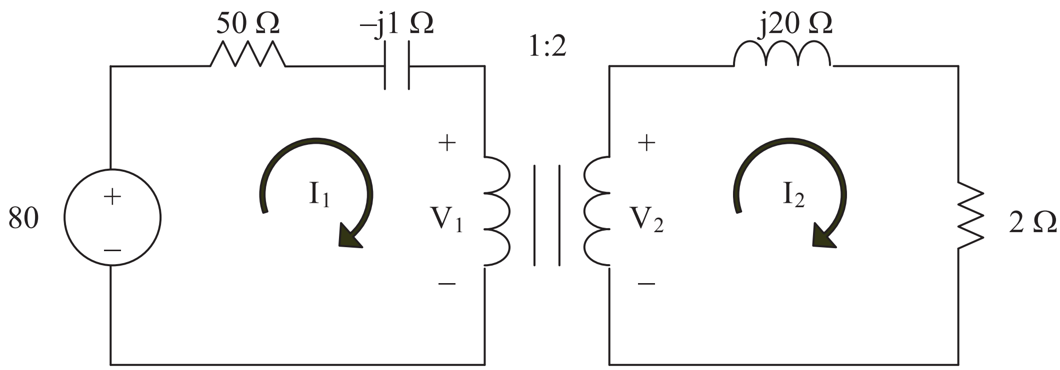
(b) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$

(c) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{1/n}$

(d) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{-1/n}$

Chapter 13, Solution 42.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-80 + (50-j)I_1 + V_1 = 0 \qquad (1)$$

For mesh 2,

$$-V_2 + (2+j20)I_2 = 0 \qquad (2)$$

At the transformer terminals,

$$V_2 = 2V_1 \text{ or } 2V_1 - V_2 = 0 \qquad (3)$$

$$I_1 = 2I_2 \text{ or } I_1 - 2I_2 = 0 \qquad (4)$$

From (1) to (4),

$$\begin{bmatrix} 50-j & 0 & 1 & 0 \\ 0 & 2+j20 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this with MATLAB,

```
>> A = [(50-j) 0 1 0;0 (2+20j) 0 -1;0 0 2 -1;1 -2 0 0]

A =

  Columns 1 through 3

  50.0000 - 1.0000i        0              1.0000
        0              2.0000 +20.0000i        0
        0                   0            2.0000
   1.0000             -2.0000                  0

  Column 4

        0
  -1.0000
  -1.0000
```

```
    0

>> B = [80;0;0;0]

B =

   80
    0
    0
    0

>> C = inv(A)*B

C =

  1.5743 - 0.1247i              (I1)
  0.7871 - 0.0623i              (I2)
  1.4106 + 7.8091i              (V1)
  2.8212 +15.6181i              (V2)
```

$$I_2 = (787.1–j62.3) \text{ mA or } 789.6\angle–4.53° \text{ mA}$$

The power absorbed by the 2-Ω resistor is

$$P = |I_2|^2R = (0.7896)^2 2 = \mathbf{1.2469\ W}.$$

Chapter 13, Solution 57.

(a) $Z_L = j3||(12 - j6) = j3(12 - j6)/(12 - j3) = (12 + j54)/17$

Reflecting this to the primary side gives

$Z_{in} = 2 + Z_L/n^2 = 2 + (3 + j13.5)/17 = 2.3168\angle 20.04°$

$I_1 = v_s/Z_{in} = 60\angle 90°/2.3168\angle 20.04° =$ **$25.9\angle 69.96°$ A(rms)**

$I_2 = I_1/n =$ **$12.95\angle 69.96°$ A(rms)**

(b) $60\angle 90° = 2I_1 + v_1$ or $v_1 = j60 - 2I_1 = j60 - 51.8\angle 69.96°$

$v_1 =$ **$21.06\angle 147.44°$ V(rms)**

$v_2 = nv_1 =$ **$42.12\angle 147.44°$ V(rms)**

$v_o = v_2 =$ **$42.12\angle 147.44°$ V(rms)**

(c) $S = v_s I_1^* = (60\angle 90°)(25.9\angle -69.96°) =$ **$1.554\angle 20.04°$ kVA**